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Safety Of Complex Technical System Impacted By Its Operation Process

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Abstract

An innovative approach is proposed to safety analysis of multistate ageing systems impacted by their operation processes. A safety function and other safety indicators are defined for a complex multistate ageing system changing its safety structure and its components safety parameters during the operation and determined under the assumption that its components have piecewise exponential safety functions. Results are applied to examine safety of port and maritime transportation systems.

Keywords: multistate system, ageing, operation process impact, safety, transport, port oil terminal, maritime ferry

1. Introduction

General approach to safety analysis of complex technical system related to its operation process is presented and applied to safety evaluation of port and maritime transportation systems, the interdependent critical infrastructures (Lague et al., 2015). The safety function and risk function for the port oil terminal and the technical system of a maritime ferry are determined (Magryta-Mut, 2023). Other determined for these two systems, practically significant safety and resilience indicators, are the mean lifetime up to the exceeding a critical safety state, the moment when the risk function value exceeds the acceptable safety level, the intensity of ageing/degradation, the coefficient of operation process impact on intensities of ageing and the coefficient of resilience to operation process impact.

2. Multistate approach to ageing system safety

Similarly, as in the case of multistate approach to system reliability (Kołowrocki, 2014), in the multistate system safety analysis to define the system with degrading/ageing components, we assume that:

- n is the number of the system components (assets);
- E_i, $i = 1,2,\ldots,n$, are the system components;
- all components and the system have the safety state set $\{0,1,\ldots,z\}$, $z \geq 1$;
- the safety states are ordered, the safety state 0 is the worst and the safety state z is the best;
- r, $r \in \{1,2,\ldots,z\}$, is the critical safety state (the system and its components staying in the safety states less than the critical state, i.e. in safety states $0,1,2,\ldots,r-1$, is highly dangerous for them and for their operating area);
- $T_i(u)$, $i = 1,2,\ldots,n$, are random variables representing the lifetimes of system components E_i in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 0,1,2,\ldots,z$, while they were in the safety state z at the moment $t = 0$;
- $T(u)$ is a random variable representing the lifetime of the system in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 0,1,2,\ldots,z$, while it was in the safety state z at the moment $t = 0$;
- the safety states degrade with time t;
- the components and the system safety states degrade with time t;
- $s_i(t)$ is the component E_i, $i = 1,2,\ldots,n$, safety state at the moment t, $t \in \langle 0,\infty)$, while it was in the safety state z at the moment $t = 0$;
- $s(t)$ is the system safety state at the moment t, $t \in \langle 0,\infty)$, given that it was in the safety state z at the moment $t = 0$.

The above assumptions mean that the safety states of the system with degrading components may be changed in time only from better to worse (Kołowrocki, 2014).

We define the system safety function by the vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \ldots, \boldsymbol{S}(t,z)] \quad (1)$$

for $t \in \langle 0,\infty)$, where

$$\boldsymbol{S}(t,u) = P(s(t) \geq u | s(0) = z) = P(T(u) > t) \quad (2)$$

for $t \in \langle 0,\infty)$, $u = 1,2,\ldots,z$, is the probability that the multistate system is in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the moment t, $t \in \langle 0,\infty)$, while it was in the safety state z at the moment $t = 0$.

We do not consider $\boldsymbol{S}(t,0)$ in (1) as

$$\boldsymbol{S}(t,0) = P(s(t) \geq 0 | s(0) = z) = P(T(0) > t) = 1$$

for $t \in \langle 0,\infty)$, what means that it is constant.

The safety functions (t,u), $t \in \langle 0,\infty)$, $u = 1,2,\ldots,z$, defined by Eq.(2) are called the coordinates of the system safety function $\boldsymbol{S}(t,\cdot)$ defined by (1). Thus, the relationship between the distribution function $\boldsymbol{F}(t,u)$, of the system lifetime $T(u)$, $u = 1,2,\ldots,z$, in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, and the coordinate $\boldsymbol{S}(t,u)$, $t \in \langle 0,\infty)$, $u = 1,2,\ldots,z$, of its safety function is given by

$$\boldsymbol{F}(t,u) = P(T(u) \leq t) = 1 - P(T(u) > t) = 1 - \boldsymbol{S}(t,u),\ t \in \langle 0,\infty),\ u = 1,2,\ldots,z.$$

The graph of an exemplary four-state (z = 3) system safety function

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3)],\ t \in \langle 0,\infty),$$

is shown in Figure 1.

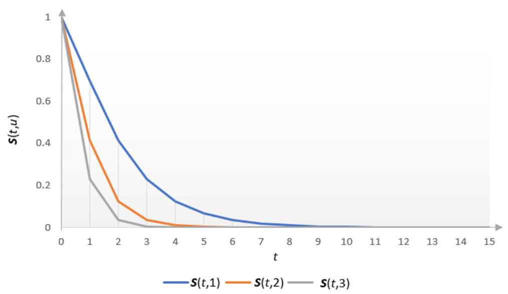

Fig. 1. The graphs of an exemplary four-state system safety function coordinates.

If r is the critical safety state, then the multistate system risk function

$$\boldsymbol{r}(t) = P(s(t) < r | s(0) = z) = P(T(r) \leq t) \quad (3)$$

for $t \in \langle 0,\infty)$, is defined as a probability that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,2,\ldots,z\}$, while it was in the best safety state z at the moment $t = 0$ and given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r),\ t \in \langle 0,\infty),\tag{4}$$

where $\boldsymbol{S}(t,r)$ is the coordinate of the multistate system safety function given by Eq.(2) for $u = r$.
The graph of the exemplary system risk function is presented in Figure 2.

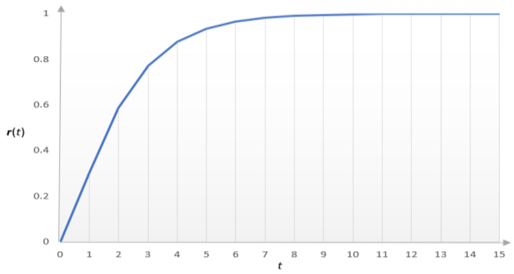

Fig. 2. The graph of an exemplary system risk function.

The moment $\boldsymbol{\tau}$, when the system risk function exceeds a permitted level δ, $\delta \in (0,1)$, is defined by

$$\boldsymbol{\tau} = \boldsymbol{r}^{-1}(\delta),\tag{5}$$

where $\boldsymbol{r}^{-1}(t)$, $t \in \langle 0,\infty)$, is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (4).
The intensities of ageing of a multistate ageing system, i.e. the intensities of system departure from the safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2,\dots,z$ are defined by

$$\boldsymbol{\lambda}(t,u) = \frac{-\frac{d\boldsymbol{S}(t,u)}{dt}}{\boldsymbol{S}(t,u)},\ t \in \langle 0,\infty),\ u = 1,2,\dots,z\tag{6}$$

where (t,u), $u = 1,2,\dots,z$, are the coordinate of the multistate ageing system safety function given by Eq.(2). Whereas, the multistate ageing system approximate mean intensities of ageing are defined by

$$\boldsymbol{\lambda}(u) = \frac{1}{\boldsymbol{\mu}(u)},\ u = 1,2,\dots,z,\tag{7}$$

where (u), $u = 1,2,\dots,z$, are the mean values of the multistate ageing system lifetimes in safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2,\dots,z$.

The coefficients of the impact on the multistate ageing system safety are defined by

$$\boldsymbol{\rho}(u) = \boldsymbol{\lambda}(u)/\boldsymbol{\lambda}^0(u),\ u = 1,2,\dots,z,\tag{8}$$

where $\boldsymbol{\lambda}(u)$ and $\boldsymbol{\lambda}^0(u)$, $u = 1,2,\dots,z$, respectively are, the intensities of ageing of the multistate ageing system with and without impact, determined according to (6) or (7).

Finally, we define the multistate ageing system resilience indicators, i.e. the coefficients of the multistate ageing system resilience to the impact, by

$$\boldsymbol{RI}(u) = \frac{1}{\boldsymbol{\rho}(u)},\ u = 1,2,\dots,z,\tag{9}$$

where $\boldsymbol{\rho}(u)$, $u = 1,2,\dots,z$, are the coefficients of the impact on the multistate ageing system safety defined by (8).

3. Safety of multistate ageing system impacted by operation process

3.1. Semi-Markov model of system operation process

We assume that the system during its operation process is taking $\nu, \nu \in N$, different operation states $z_1, z_2, \dots, z_\nu$. Further, we define the system operation process $Z(t)$, $t \in \langle 0,\infty)$, with discrete operation states from the set $\{z_1, z_2, \dots, z_\nu\}$.
Moreover, we assume that the system operation process $Z(t)$ is a semi-Markov process (Grabski, 2014), (Kołowrocki, 2014) with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,\dots,\nu$, $b \neq l$. Under these assumptions, the system operation process may be described by the following parameters:

- the vector $[p_b(0)]_{1x\nu}$ of the initial probabilities $p_b(0) = P(Z(0) = z_b$, $b = 1,2,\dots,\nu$, of the system operation process Z(t) staying at particular operation states at the moment $t = 0$;
- the matrix $[p_{bl}]_{\nu x\nu}$ of probabilities p_{bl}, $b, l = 1,2,\dots,\nu$, $b \neq l$ of the system operation process Z(t) transitions between the operation states z_b and z_l;

- the matrix $[H_{bl}(t)]_{\nu x\nu}$ of conditional distribution functions $H_{bl}(t) = P(\theta_{bl} < t)$, $t \in \langle 0, \infty)$, $b, l = 1,2,\ldots,\nu$, $b \neq l$, of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states or quivalently by the matrix $[h_{bl}(t)]_{\nu x\nu}$ of the conditional density functions $h_{bl}(t)$, $t \in \langle 0, \infty)$, $b, l = 1,2,\ldots,\nu$, $b \neq l$, of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states corresponding to the conditional distribution functions $H_{bl}(t)$.

The knowledge of the system operation process parameters gives the possibility of finding its main characteristics:

- the mean values $M_b = E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,\ldots,\nu$, at the operation states

$$M_b = \sum_{l=1}^{\nu} p_{bl} M_{bl}, \; b = 1,2,\ldots,\nu, \tag{10}$$

where $M_{bl} = E[\theta_{bl}]$ are the mean values of the conditional sojourn times θ_{bl} at the operation states;

- the limit values $p_b = \lim_{t\to\infty} p_b(t)$ of the system operation process $Z(t)$ transient probabilities at the particular operation states $p_b(t) = P(Z(t) = z_b)$, $t \in \langle 0, \infty)$, $b = 1,2,\ldots,\nu$, (Kołowrocki, 2014)

$$p_b = \frac{\pi_b M_b}{\sum_{l=1}^{\nu} \pi_l M_l}, \; b = 1,2,\ldots,\nu, \tag{11}$$

where π_b are the steady probabilities of the vector $[\pi_b]_{1x\nu}$,satisfying the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{\nu} \pi_l = 1; \end{cases}$$

- the approximate mean values $\hat{M}_b = E[\hat{\theta}_b]$ of the system operation process $Z(t)$ total sojourn times θ_b at the particular operation states z_b, $b = 1,2,\ldots,\nu$, during the large fixed system opetation time θ (Grabski, 2014; Kołowrocki, 2014)

$$\hat{M}_b = p_b \theta, \; b = 1,2,\ldots,\nu. \tag{12}$$

3.2. Safety of system related to its operation process

We assume that the changes of the operation states of the system operation process $Z(t)$ have an influence on the safety of system multistate components E_i, $i = 1,2,\ldots,n$, sand the system safety structure as well. Consequently, we denote the system multistate component E_i, $i = 1,2,\ldots,n$, conditional lifetime in the safety state subset $\{u, u+1, \ldots, z\}$ while the system is at the operation state z_b, $b = 1,2,\ldots,\nu$, by $[T_i(u)]^{(b)}$ and its conditional safety function by the vector

$$[S_i(t,\cdot)]^{(b)} = [S_i(t,1)]^{(b)}, \ldots, [S_i(t,z)]^{(b)}], \tag{13}$$

with the coordinates defined by

$$[S_i(t,u)]^{(b)} = P([T_i(u)]^{(b)} > t \mid Z(t) = z_b)$$

for $\in \langle 0, \infty)$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu$.

The safety function $[S_i(t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $[T_i(u)]^{(b)}$ in the safety state subset $\{u, u+1, \ldots, z\}$ is greater than *t*, while the system operation process $Z(t)$ is at the operation state z_b.

Similarly, we denote the system conditional lifetime in the safety state subset $\{u, u+1, \ldots, z\}$ while the system is at the operation state z_b, $b = 1,2,\ldots,\nu$, by $[T(u)]^{(b)}$and the conditional safety function of the system by the vector

$$[\boldsymbol{S}(t,\cdot)]^{(b)} = [\boldsymbol{S}(t,1)]^{(b)}, \ldots, [\boldsymbol{S}(t,z)]^{(b)}], \tag{14}$$

with the coordinates defined by

$$[\boldsymbol{S}(t,u)]^{(b)} = P([T(u)]^{(b)} > t | Z(t) = z_b)$$

for $\in \langle 0, \infty)$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu$.

The safety function $[\boldsymbol{S}(t,u)]^{(b)}$is the conditional probability that the system lifetime $[T(u)]^{(b)}$ in the safety state subset $\{u, u+1, \ldots, z\}$ is greater than *t*, while the system operation process $Z(t)$ is at the operation state z_b. Thus, the system conditional lifetimes in the safety states subset $\{u, u+1, \ldots, z\}$ at the operational state z_b

$$[T(u)]^{(b)} = T([T_1(u)]^{(b)}, [T_2(u)]^{(b)}, \ldots, [T_n(u)]^{(b)})$$

defined for $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu$, $n \in N$ are dependent on the components conditional lifetimes $[T_1(u)]^{(b)}$, $[T_2(u)]^{(b)}$,…, $[T_n(u)]^{(b)}$, in the safety states subset $\{u, u+1, \ldots, z\}$ at the operation state z_b and consequently, the coordinates of the system conditional safety function at the operation state z_b

$$[\boldsymbol{S}(t,u)]^{(b)} = \boldsymbol{S}([S_1(t,u)]^{(b)}, [S_2(t,u)]^{(b)}, \dots, [S_n(t,u)]^{(b)}) \tag{14}$$

defined for $t \in \langle 0,\infty)$, $u = 1,2,\dots,z$, $b = 1,2,\dots,\nu$, $n \in N$ are dependent on the coordinates

$$[S_1(t,u)]^{(b)}, [S_2(t,u)]^{(b)}, \dots, [S_n(t,u)]^{(b)}$$

of the components conditional safety functions at the operation state z_b, defined by Eq.(13).

Further, we denote the system unconditional lifetime in the safety state subset $\{u, u+1, \dots, z\}$ by $T(u)$ and the unconditional safety function of the system by the vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \dots, \boldsymbol{S}(t,z)],\ t \in \langle 0,\infty), \tag{15}$$

with the coordinates defined by

$$\boldsymbol{S}(t,u) = P(T(u) > t) \text{ for } t \in \langle 0,\infty),\ u = 1,2,\dots,z.$$

In the case when the system operation time θ is large enough, the coordinates of the unconditional safety function of the system defined by Eq.(13) are given by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{S}(t,u)]^{(b)}, \tag{16}$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $t \in \langle 0,\infty)$, $u = 1,2,\dots,z$, $b = 1,2,\dots,\nu$, are the coordinates of the system conditional safety functions defined by Eq.(13) and (14) and p_b, $b = 1,2,\dots,\nu$, are the system operation process limit transient probabilities given by (8).

The mean value of the system unconditional lifetime $T(u)$ in the safety state subset $\{u, u+1, \dots, z\}$ is given by (Kołowrocki, 2014)

$$\boldsymbol{\mu}(u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{\mu}(u)]^{(b)},\ u = 1,2,\dots,z, \tag{17}$$

where $[\boldsymbol{\mu}(u)]^{(b)}$, are the mean values of the system conditional lifetimes $[T(u)]^{(b)}$ in the safety state subset $\{u, u+1, \dots, z\}$ at the operation state z_b, $b = 1,2,\dots,\nu$, given by

$$[\boldsymbol{\mu}(u)]^{(b)} = \int_0^\infty [\boldsymbol{S}(t,u)]^{(b)} dt,\ u = 1,2,\dots,z, \tag{18}$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,\dots,z$, $b = 1,2,\dots,\nu$, are defined by (13) and (14) and p_b are given by (8). Whereas, the variance of the system unconditional lifetime $T(u)$ is given by

$$\sigma^2(u) = 2\int_0^\infty t\, \boldsymbol{S}(t,u) dt - [\boldsymbol{\mu}(u)]^2,\ u = 1,2,\dots,z, \tag{19}$$

where $\boldsymbol{S}(t,u)$, $u = 1,2,\dots,z$, are given by (15) and (16) and $\boldsymbol{\mu}(u)$, $u = 0,1,\dots,z$, are given by (17) and (18).
Hence, we get the following formulae for the mean values of the unconditional lifetimes of the system in particular safety states

$$\bar{\boldsymbol{\mu}}(u) = \boldsymbol{\mu}(u) - \boldsymbol{\mu}(u+1),\ u = 1,2,\dots,z-1,\ \bar{\boldsymbol{\mu}}(z) = \boldsymbol{\mu}(z), \tag{20}$$

where $\boldsymbol{\mu}(u)$, $u = 1,2,\dots,z$, are given by (17) and (18).
Moreover, if r is the system critical safety state, then the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r),\ t \in \langle 0,\infty), \tag{21}$$

where $\boldsymbol{S}(t,r)$ is the coordinate of the system unconditional safety function given by (16) for $u = r$ and if τ is the moment when the system risk function exceeds a permitted level δ, then

$$\boldsymbol{\tau} = \boldsymbol{r}^{-1}(\delta), \tag{22}$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (21).

The intensities of degradation (ageing) of the critical infrastructure, i.e. the intensities of the critical infrastructure departure from the safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2,\dots,z$, are

$$\boldsymbol{\lambda}(t,u) = \frac{-\frac{d\boldsymbol{S}(t,u)}{dt}}{\boldsymbol{S}(t,u)},\ t \in \langle 0,\infty),\ u = 1,2,\dots,z, \tag{23}$$

where $\boldsymbol{S}(t,u)$, $t \in \langle 0,\infty)$, $u = 1,2,\dots,z$, are defined by (16).

Next, we define the coefficients of operation process impact on the system intensities of degradation (the intensities of departure from the safety state subset $\{u, u+1, \dots, z\}$, $u = 1,2,\dots,z$) by

$$\boldsymbol{\rho}(t,u) = \frac{\boldsymbol{\lambda}(t,u)}{\boldsymbol{\lambda}^0(t,u)},\ t \in \langle 0,\infty) \text{ or } \boldsymbol{\rho}(u) = \frac{\boldsymbol{\lambda}(u)}{\boldsymbol{\lambda}^0(u)}, \tag{24}$$

for $u = 1,2,\dots,z$, where $\boldsymbol{\lambda}^0(t,u)$, $t \in \langle 0,\infty)$ and $\boldsymbol{\lambda}^0(u)$, $u = 1,2,\dots,z$, are the intensities of degradation of the system without of operation process impact, whereas $\boldsymbol{\lambda}(t,u)$, $t \in \langle 0,\infty)$ and $\boldsymbol{\lambda}(u)$, $u = 1,2,\dots,z$, are the intensities of degradation of the system with the operation process impact determined by (23).
The indicators of system resilience to operation process impact are defined by

$$\boldsymbol{RI}(t,u) = \frac{1}{\boldsymbol{\rho}(t,u)}, t \in \langle 0,\infty) \text{ or } \boldsymbol{RI}(u) = \frac{1}{\boldsymbol{\rho}(u)}, \tag{25}$$

for $u = 1,2,\dots,z$, where $\boldsymbol{\rho}(t,u)$, $t \in \langle 0,\infty)$ and $\boldsymbol{\rho}(u)$, $u = 1,2,\dots,z$, are the coefficients of operation process impact on the system intensities od degradation given by (24).

Further, we assume that the system components E_i, $i = 1,2,\dots,n$, conditional safety functions at the system operation states z_b, $b = 1,2,\dots,\nu$, defined by (13), are piecewise exponential, i.e. their coordinates are given by

$$[S_i(t,u)]^{(b)} = \exp\left[-[\lambda_i(u)]^{(b)} t\right] \tag{26}$$

for $t \in \langle 0,\infty)$, $i = 1,2,\dots,n$, $u = 1,2,\dots,z$, $b = 1,2,\dots,\nu$, and we conclude that the system conditional safety functions defined by Eq.(14) are dependent of these piecewise exponential safety functions.

4. Applications

4.1. Safety of port oil terminal critical infrastructure

We consider the port oil terminal critical infrastructure impacted by its operation process placed at the Baltic seaside that is designated for receiving oil products from ships, storage and sending them by carriages or trucks.

Fig.3. The port oil terminal critical infrastructure operating area.

The main technical assets (components) of the port oil terminal critical infrastructure are:

- A_1 – port oil piping transportation system;
- A_2 – internal pipeline technological system;
- A_3 – supporting pump station;
- A_4 – internal pump system;
- A_5 – port oil tanker shipment terminal;
- A_6 – loading railway carriage station;
- A_7 – loading road carriage station;
- A_8 – unloading railway carriage station;
- A_9 – oil storage reservoir system.

We distinguish the following three safety states ($z = 2$) of the system and its components:

- a safety state 2 – the components and the port oil terminal are fully safe;
- a safety state 1 – the components and the port oil terminal are less safe and more dangerous because of the possibility of environment pollution;
- a safety state 0 – the components and the port oil terminal are destroyed.

The port oil terminal system safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2)], t \in \langle 0,\infty), \tag{27}$$

with the coordinates (Magryta-Mut, 2023):

$$\boldsymbol{S}(t,1) = 0.395\cdot[\boldsymbol{S}(t,1)](1) + 0.060\cdot[\boldsymbol{S}(t,1)](2) + 0.003\cdot[\boldsymbol{S}(t,1)](3) + 0.002\cdot[\boldsymbol{S}(t,1)](4) + 0.2\cdot[\boldsymbol{S}(t,1)] + 0.058\cdot[\boldsymbol{S}(t,1)](6) + 0.282\cdot[\boldsymbol{S}(t,1)], \tag{28}$$

$\boldsymbol{S}(t,2) = 0.395\cdot[\boldsymbol{S}(t,2)](1) + 0.060\cdot[\boldsymbol{S}(t,2)](2) + 0.003\cdot[\boldsymbol{S}(t,2)](3) + 0.002\cdot[\boldsymbol{S}(t,2)](4) + 0.2\cdot[\boldsymbol{S}(t,2)]$
$+ 0.058\cdot[\boldsymbol{S}(t,2)](6) + 0.282\cdot[\boldsymbol{S}(t,2)].$ (29)

The graph of this three-state port oil terminal safety function is shown in Figure 4.

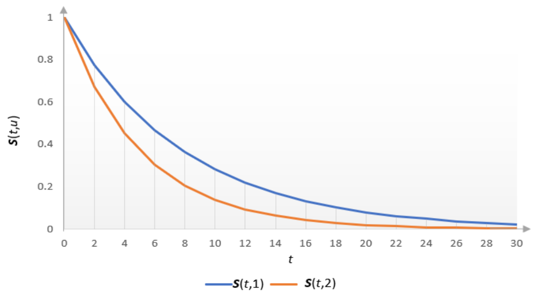

Fig. 4. The graph of the port oil terminal safety function coordinates.

The expected values and standard deviations of the terminal lifetimes in the safety state subsets {1, 2}, {2}, in years, after applying (17)-(19) and (28)-(29), respectively, are (Magryta-Mut, 2023):

$\boldsymbol{\mu}(1) \cong 7.89,\ \boldsymbol{\mu}(2) \cong 5.03,$ (30)

$\boldsymbol{\sigma}(1) \cong 7.91, \boldsymbol{\sigma}(2) \cong 5.03,$ (31)

and further, from (30), by (20), the expected values of the terminal lifetimes in the particular safety states {1}, {2}, in years, respectively, are

$\overline{\boldsymbol{\mu}}(1) \cong 2.86,\ \overline{\boldsymbol{\mu}}(2) \cong 5.03.$ (32)

Since the critical safety state is $r = 1$, then according to (21), the terminal risk function is given by

$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,1)$ for $t \geq 0,$ (33)

where $\boldsymbol{S}(t,1)$ is given by (28).
The graph of the port oil terminal risk function is presented in Figure 5.

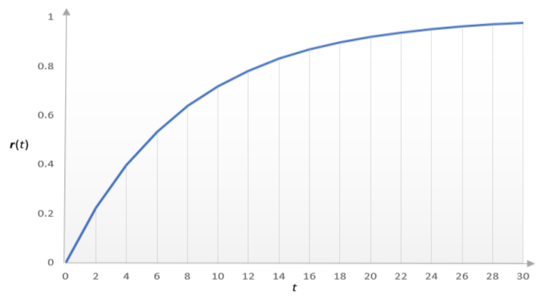

Fig. 5. The graph of the port oil terminal risk function.

The moment τ, when system risk function exceeds a permitted level $\delta = 0.05$, according to (22) and (33), is (Magryta-Mut, 2023)

$\boldsymbol{\tau} = \boldsymbol{r}^{-1}(\delta) \cong 0.40$ year. (34)

The port oil terminal critical infrastructure approximate mean intensities of ageing, according to (9) and (32), are:

$\boldsymbol{\lambda}(1) \cong 0.126743,\ \boldsymbol{\lambda}(2) \cong 0.198807.$ (35)

The coefficients of the operation process impact on the port oil terminal critical infrastructure intensities of ageing, are:

$\boldsymbol{\rho}(1) \cong 1.09381,\ \boldsymbol{\rho}(2) \cong 1.09391.$ (36)

Finally, the port oil terminal resilience indicator, i.e. the coefficient of the port oil terminal resilience to the operation process impact, for $u = r = 1$, is

$\boldsymbol{RI}(1) \cong 0.9142 = 91.42\%.$ (37)

4.2. Safety of maritime ferry technical system

The considered maritime ferry is a passenger ship operating at the Baltic Sea between Gdynia and Karlskrona ports on regular everyday line.

Fig. 6. The maritime ferry operating area.

The ferry technical system is composed of the following subsystems:

- S_1 – a navigational subsystem;
- S_2 – a propulsion and controlling subsystem;
- S_3 – a loading and unloading subsystem;
- S_4 – a stability control subsystem;
- S_5 – an anchoring and mooring subsystem.

We identify the five safety states of the ferry technical system and its components:

- a safety state 4 – the ferry operation is fully safe;
- a safety state 3 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution;
- a safety state 2 – the ferry operation is less safe and more dangerous because of the possibility of environment pollution and causing small accidents;
- a safety state 1 – the ferry operation is much less safe and much more dangerous because of the possibility of serious environment pollution and causing extensive accidents;
- a safety state 0 – the ferry technical system is destroyed.

The maritime technical system safety function is given by the vector (Magryta-Mut, 2023)

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)] \text{ for } t \in \langle 0, \infty), \quad (38)$$

with the coordinates:

$$\boldsymbol{S}(t,1) = 0.038\cdot[\boldsymbol{S}(t,1)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(2)} + 0.026\cdot[\boldsymbol{S}(t,1)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,1)]^{(4)} + 0.363\cdot[\boldsymbol{S}(t,1)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,1)]^{(6)} + 0.005\cdot[\boldsymbol{S}(t,1)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,1)]^{(8)} + 0.037\cdot[\boldsymbol{S}(t,1)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(10)} + 0.003\cdot[\boldsymbol{S}(t,1)]^{(11)} + 0.016\cdot[\boldsymbol{S}(t,1)]^{(12)} + 0.351\cdot[\boldsymbol{S}(t,1)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,1)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,1)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,1)]^{(16)} + 0.005\cdot[\boldsymbol{S}(t,1)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,1)]^{(18)}, \ t \in \langle 0, \infty), \quad (39)$$

$$\boldsymbol{S}(t,2) = 0.038\cdot[\boldsymbol{S}(t,2)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(2)} + 0.026\cdot[\boldsymbol{S}(t,2)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,2)]^{(4)} + 0.363\cdot[\boldsymbol{S}(t,2)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,2)]^{(6)} + 0.005\cdot[\boldsymbol{S}(t,2)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,2)]^{(8)} + 0.037\cdot[\boldsymbol{S}(t,2)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(10)} + 0.003\cdot[\boldsymbol{S}(t,2)]^{(11)} + 0.016\cdot[\boldsymbol{S}(t,2)]^{(12)} + 0.351\cdot[\boldsymbol{S}(t,2)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,2)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,2)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,2)]^{(16)} + 0.005\cdot[\boldsymbol{S}(t,2)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,2)]^{(18)}, \ t \in \langle 0, \infty), \quad (40)$$

$$\boldsymbol{S}(t,3) = 0.038\cdot[\boldsymbol{S}(t,3)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(2)} + 0.026\cdot[\boldsymbol{S}(t,3)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,3)]^{(4)} + 0.363\cdot[\boldsymbol{S}(t,3)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,3)]^{(6)} + 0.005\cdot[\boldsymbol{S}(t,3)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,3)]^{(8)} + 0.037\cdot[\boldsymbol{S}(t,3)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(10)} + 0.003\cdot[\boldsymbol{S}(t,3)]^{(11)} + 0.016\cdot[\boldsymbol{S}(t,3)]^{(12)} + 0.351\cdot[\boldsymbol{S}(t,3)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,3)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,3)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,3)]^{(16)} + 0.005\cdot[\boldsymbol{S}(t,3)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,3)]^{(18)}, \ t \in \langle 0, \infty), \quad (41)$$

$$\boldsymbol{S}(t,4) = 0.038\cdot[\boldsymbol{S}(t,4)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(2)} + 0.026\cdot[\boldsymbol{S}(t,4)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,4)]^{(4)} + 0.363\cdot[\boldsymbol{S}(t,4)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,4)]^{(6)} + 0.005\cdot[\boldsymbol{S}(t,4)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,4)]^{(8)} + 0.037\cdot[\boldsymbol{S}(t,4)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(10)} + 0.003\cdot[\boldsymbol{S}(t,4)]^{(11)} + 0.016\cdot[\boldsymbol{S}(t,4)]^{(12)} + 0.351\cdot[\boldsymbol{S}(t,4)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,4)]^{(14)} + 0.024\cdot[\boldsymbol{S}(t,4)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,4)]^{(16)} + 0.005\cdot[\boldsymbol{S}(t,4)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,4)]^{(18)}, \ t \in \langle 0, \infty). \quad (42)$$

The graph of this five-state ferry technical system safety function is shown in Figure 7.

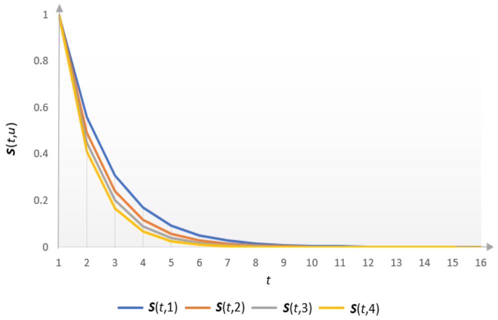

Fig. 7. The graph of the ferry technical system safety function coordinates.

The mean values and standard deviations of the ferry technical system lifetimes in the safety state subsets {1, 2, 3, 4}, {2, 3, 4}, {3, 4}, {4}, expressed in years, after applying (17)-(19) and (39)-(42), are (Magryta-Mut, 2023):

$$\boldsymbol{\mu}(1) \cong 1.694, \boldsymbol{\mu}(2) \cong 1.395, \boldsymbol{\mu}(3) \cong 1.244, \boldsymbol{\mu}(4) \cong 1.114, \quad (43)$$

$$\boldsymbol{\sigma}(1) \cong 1.669, \boldsymbol{\sigma}(2) \cong 1.396, \boldsymbol{\sigma}(3) \cong 1.230, \boldsymbol{\sigma}(4) \cong 1.102 \quad (44)$$

and further, from (43), by (20), the expected values of the ferry technical system lifetimes in the particular safety states {1}, {2}, {3}, {4}, in years, respectively, are

$$\bar{\boldsymbol{\mu}}(1) \cong 0.299, \bar{\boldsymbol{\mu}}(2) \cong 0.151, \bar{\boldsymbol{\mu}}(3) \cong 0.130, \bar{\boldsymbol{\mu}}(4) \cong 1.114. \quad (45)$$

As the critical safety state is r =2, then according to (21), the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t, 2), \text{ for } t \geq 0, \quad (46)$$

where $\boldsymbol{S}(t, 2)$ is given by (40).
The graph of the ferry technical system risk function is presented in Figure 8.

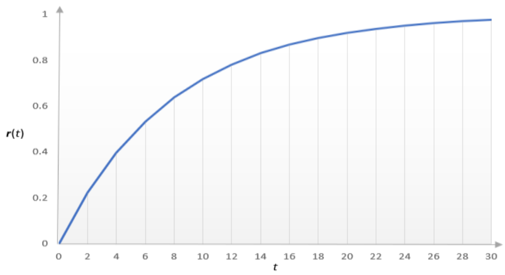

Fig. 8. The graph of the ferry technical system risk function.

The moment $\boldsymbol{\tau}$, when system risk function exceeds a permitted level $\delta = 0.05$, is

$$\boldsymbol{\tau} = \boldsymbol{r}^{-1}(\delta) = 0.073 \text{ year}. \quad (46)$$

The ferry technical system approximate mean intensities of ageing are:

$$\boldsymbol{\lambda}(1) \cong 0.590363, \boldsymbol{\lambda}(2) \cong 0.716869, \boldsymbol{\lambda}(3) \cong 0.803573, \boldsymbol{\lambda}(4) \cong 0.897470. \quad (47)$$

The coefficients of the operation process impact on the ferry technical system intensities of ageing, are:

$$\boldsymbol{\rho}(1) \cong 1.044942, \boldsymbol{\rho}(2) \cong 1.058098, \boldsymbol{\rho}(3) \cong 1.044645, \boldsymbol{\rho}(4) \cong 1.044655. \quad (48)$$

Finally, the ferry technical system resilience indicator, i.e. the coefficient of the ferry technical system resilience to the operation process impact, for $u = r = 2$, is

$$\boldsymbol{RI}(2) \cong 0.9451 = 94.51\%. \quad (49)$$

Summary

As a consequence of the achieved new results, the further research could be focused on safety analysis of multistate ageing complex systems (Kołowrocki, 2014) and critical infrastructure networks (Lague et al., 2015), considering their ageing (Szymkowiak, 2019), inside dependencies (Kołowrocki, 2020), outside impacts, including separate and joint operation and climate-weather change impacts (Kołowrocki, 2021) and the use of the achieved results to improve their safety (Magryta-Mut, 2023), strengthen their resilience and mitigate the effects of their degradation and failures (Bogalecka, 2020).

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