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Assessment Of Bivariate Process-Based Mean Residual Time Model: Application To Maintenance Planning In Parallel System

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Classical mean residual time models have been key to effectively plan different maintenance actions on a system. In general terms, mean residual time at time t provides information about the expected duration left before the system fails given the current state of the system at time t. The mean residual time depends not only on time t but also on level of the available information on the components state (or covariates state) at time t (Do and Berenger, 2021).

In this work, the maintenance scheme of a parallel system consisting of n identical and independent components and working in a random environment is analyzed. Let $X(t)$ be the environmental state at time t. We assume that the system works under the environmental state 0 (normal environmental state) or under the state 1 (severe environmental state).

Let τ be the time at which the environment shifts to a severe state

$$\tau = \inf\{t \geq 0, X(t)=1\}.$$

The deterioration of each component of the system is induced by its age and by the effect of the environment. So, the failure point process of each component is given by

$$r(t) = r_0(t)\mathbf{1}(\tau > t) + r_1(t)\mathbf{1}_{(\tau \leq t)}, \quad t \geq 0,$$

where $\mathbf{1}$ denotes the indicator function, r_0 and r_1 stand for two continuous and non-decreasing failure rates: $r_i(t)$ is the hazard rate of a component associated with the environmental state i ($i = \{0,1\}$) with $r_0(t) \leq r_1(t)$ for all t. The expression of $r(t)$ means that the environment affects to the occurrence of the failures, that is, the component is more prone to failure under a severe environmental state. It can be checked that the failure rate function of each component is increasing failure rate under some conditions. For a parallel system with identical and independent components, if the failure times of the components are increasing failure rates then the failure time of the system is increasing failure rate (Samaniego, 1985).

On the other hand, we assume that when the number of failed components exceeds d (with $d < n$), the system is less efficient and it causes losses of production (Santos and Cavalcante, 2018). In connection to the delay-time framework (Christer 1999), the system condition is divided into three states.

1. If the number of failed components is less than d, the system is properly working.
2. If the number of failed components is greater or equal to d but less than n, the system works with a reduced performance.
3. When the number of failed components is n, the system is failed.

Under this framework, a new index called bivariate state dependent mean residual time is proposed to plan different maintenance actions. For that, the system is inspected periodically. The information available on the system in these inspection times is the number of failed components and the environmental state in which the system is working. Given this information, the expected residual time is considered as the expected time from this inspection time until the failure of the d-th component (with $d < n$). We assume that the system is inspected at times $T, 2T, 3T, \ldots$. That is, if the system is inspected at time kT, the bivariate state dependent mean residual time (SDMRT) is given by

$$m_{ij}(kT) = E[T_{[d]}-kT|(N(kT) = i, X(kT) = j)],$$

where $N(kT)$ is the number of failed components at time kT and $T_{[d]}$ the time of the d-th failure of a component. Since the state-dependent mean residual time to reach the defective state is less when the system is working under a severe state, two control levels are imposed depending on the environmental state (normal or severe) in the inspection times.

Environmental state 0

- the system is left as it is when the SDMRT is greater than L_0 ($L_0 > 0$);
- the system is preventively maintained if the SDMRT is less than L_0 and the number of failed components is less than n;
- the system is correctively maintained if the number of failed components is n.

Environmental state 1

- the system is left as it is when the SDMRT is greater than L_1 ($L_1 > 0$ and $L_1 \leq L_0$);
- the system is preventively maintained if the SDMRT is less than L_1 and the number of failed components is less than n;
- the system is correctively maintained if the number of failed components is n.

Assuming a sequence of costs for the preventive maintenance actions, for the corrective maintenance actions, for the downtime and for the time spent in a defective state, an optimal maintenance policy is analyzed. For that, the expected cost rate ($C_\infty(T,L_0,L_1)$) is evaluated. The optimal maintenance policy is obtained as

$$C_\infty(T_{\text{opt}},L_{\text{opt},0},L_{\text{opt},1})=\inf\{C_\infty(T,L_0,L_1),\ T > 0,\ 0 \leq L_1 \leq L_0\},$$

that is, the parameters to optimize are the time between inspections and the preventive thresholds L_0 and L_1. This optimization problem is solved by using genetic algorithms.

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