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Importance Measures For Fault Tree Analysis Using Dynamic And Dependent Tree Theory, D^2T^2

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Dynamic and Dependent Tree Theory (Andrews and Tolo, 2023) was introduced to overcome some of the restrictive assumptions of conventional fault tree analysis. Dependent basic events, events whose failure or repair times could be of any distribution, and systems with complex maintenance strategies can be analysed using this new framework. This paper shows how the methodology can be extended to calculate the component importance measures: Birnbaum, Criticality, Risk Assessment Worth and Risk Reduction Worth.

A key feature of the D^2T^2 algorithm is that it breaks the original fault tree problem down, accounting for the dependencies that exist between events, into a series of sub-problems which are all mutually independent but may contain the dependencies and complexities. This modularization process leaves the problem in a hierarchical structure such as that illustrated in Figure 1.

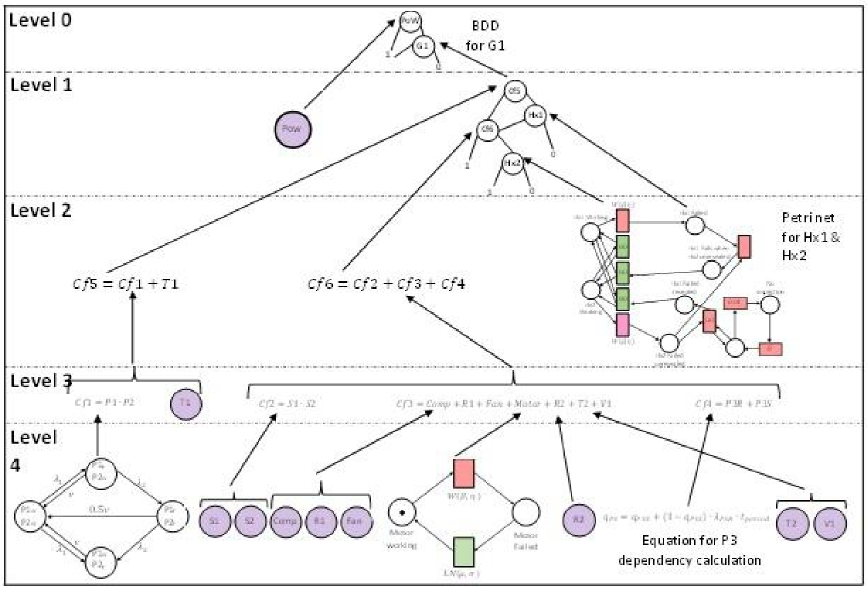

Fig. 1. Modularised Fault Tree.

Each of the modules is either a fault tree section (to be analysed using a BDD), a complex factor of independent events, or a Petri Net or Markov module containing the dependencies.

Once each module is quantified the results are integrated by taking advantage of the BDD characteristics to get the top event probability or frequency.

The following Importance measures (Rausand and Hoyland, 2004) are calculated:

Birnbaum: also known as the Criticality Function, $G_i(\boldsymbol{q(t)})$, is the probability that the system is in a critical state for component, i, where the failure of the component will cause the system to transition from the working to the failed state. This can be calculated as shown in equation 1.

$$G_i(\boldsymbol{q(t)}) = Q_{sys}(1_i, \boldsymbol{q(t)}) - Q_{sys}(0_i, \boldsymbol{q(t)}) \quad (1)$$

Criticality: is the probability that the system is in a critical state for component ***i*** and component ***i*** fails weighted according to the system failure probability.

Risk Assessment Worth: calculates the relative increase in the system unavailability when it is known that component ***i*** has failed , as in equation 2.

$$I_i^{RAW} = \frac{Q_{sys}(1_i,\boldsymbol{q(t)}) - Q_{sys}(\boldsymbol{q(t)})}{Q_{sys}(\boldsymbol{q(t)})} \quad (2)$$

where, $Q_{sys}(\boldsymbol{q(t)})$ is the system unavailability function, the vector of component unavailability's $\boldsymbol{q(t)} = (q_1(t), q_2(t), \ldots \ldots q_i(t) \ldots., q_n(t))$, $(1_i, \boldsymbol{q(t)}) = (q_1(t), q_2(t), \ldots \ldots 1_i \ldots., q_n(t))$ and $(0_i, \boldsymbol{q(t)}) = (q_1(t), q_2(t), \ldots \ldots 0_i \ldots., q_n(t))$.

Risk Reduction Worth: calculates the relative reduction in the system unavailability when it is known that component i is working. It is calculated using equation 3.

$$I_i^{RRW} = \frac{Q_{sys}(\boldsymbol{q(t)}) - Q_{sys}(0_i,\boldsymbol{q(t)})}{Q_{sys}(\boldsymbol{q(t)})} \quad (3)$$

For this particular collection of importance measures, it can be seen that they are closely related. It is necessary to calculate $Q_{sys}(1_i, \boldsymbol{q(t)})$ and $Q_{sys}(0_i, \boldsymbol{q(t)})$. These are obtained by setting the component, ***i***, to the failed and then working state and then evaluating the top event probability from the modularised structure resulting from the application of D^2T^2. The process is efficient since it reuses the probability of failure of each of the modules established when applying D^2T^2 to deliver the top event failure probability $Q_{sys}(\boldsymbol{q(t)})$.

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