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# Information-Based Maintenance Policies For Items From Heterogeneous Populations

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Optimal preventive maintenance (PM) of technical items is one of the most important practical and theoretical problems in modern reliability engineering. The first mathematically formulated results describing the optimal PM strategy can be found in the seminal paper by Barlow and Hunter (1960). After that, thousands of papers and a number of surveys and books entirely devoted to this topic (Asadi et al., 2023; Gertsbakh, 2005; Nakagawa, 2005; Wang, 2002; Wang and Pham, 2006), to name a few; were published. PM is usually reasonable to execute for deteriorating items (e.g., with increasing failure rate). When deterioration processes in items are observable, this additional information on the items' states can be used in optimal preventive maintenance decisions (thus, condition-based). See, e.g., some relevant surveys on condition-based maintenance; e.g. see (Alaswad and Xiang, 2017; Jardine and Banevich, 2005; Keiser et al., 2017) among others.

Most real populations of technical items are heterogeneous. In reliability context, items with different quality may be manufactured due to, e.g., defective resources and components, human errors, 'random' production environment, etc. (Finkelstein and Cha, 2013; Jensen and Petersen, 1982). For modeling heterogeneous lifetimes, the distribution function of an item is often indexed by an unobserved random variable $Z \geq 0$ that is called "frailty" (Badia et al., 2002; Vaupel et al., 1978). See also (Cha and Finkelstein, 2014) for some examples of frailty modeling in reliability analysis and (Finkelstein and Cha, 2013) for basics of stochastic description for heterogeneous populations. Thus, as subpopulations are ordered (see the next section for relevant definitions), the lifetime of an item from a population for replacement can be stochastically larger than the remaining lifetime of an operating item even without the assumption of increasing failure rate (e.g., for constant baseline failure rate!) This can be easily seen for the multiplicative mixing model (Finkelstein, 2008; Finkelstein and Cha, 2013) when the failure rate of a subpopulation is given by the product $z\lambda$. Indeed, when $Z = z_1$ for the operating item and $Z = z_2$ for the replacement, whereas $z_2 < z_1$, the replacement item is 'better' than the initial one. However, a natural question arises: how can we compare these realizations in practice, as the frailty is unobserved!? The answer to this question lies in using some additional information in our modeling.

In our paper we will consider a generalization of the periodic PM modeling (Nakagawa, 2005; Cha and Finkelstein, 2018). An item is replaced periodically with period $T$. The optimal $T$ is obtained as the value minimizing the corresponding expected long-run cost rate. Between replacements, the minimal repairs that do not change the failure rate of an operating item are performed. In our approach, the a'priori numbers of minimal repairs in this interval provide information on realization of frailty $Z$. The large number of minimal repairs indicates that the corresponding failure rate is relatively large and vice versa. The proposed procedure is sequential. On the first stage, the optimal $T$ is obtained in a conventional way without considering the number of minimal repairs. On the second stage, two possible values of replacement times are considered, i.e., $T_1 < T_2$ and the number of minimal repairs at $T_1$. If the number of minimal repairs observed in operation is larger than a predetermined threshold value, the replacement is performed immediately at $T_1$, whereas if it is smaller, the replacement is performed at $T_2$. Then the optimal values are obtained, via considering the corresponding optimization problem. In a similar manner, the next stage (involving $T_1, T_2, T_3$ and the numbers of minimal repairs in $[0, T_1)$ and $[T_1, T_2,))$ is defined, etc.

Thus, starting with the second stage, each stage, depending on the observed numbers of minimal repairs, can be considered as a 'multi-option' for a possible postponement of a replacement. Thus, e.g., on stage 3, the replacement is performed either at optimal $T_1$ or postponed to $T_2$ or to $T_3 > T_2$. From general considerations and it will be shown numerically that this procedure quickly converges and usually, in practice, it is sufficient to consider stage 2 or stage 3 for defining the optimal PM, as the inputs of the next stages, as compared with the previous ones are practically non-significant. A similar notion of information-based possible postponement of replacement was considered recently in (Finkelstein et al., 2023). However, information in this paper was in the form of the observed at inspections degradation of an item modeled by the Poisson or gamma processes, which dramatically differs from the heterogeneous setting of a current paper. On the other hand, periodic replacement policy for the discrete heterogeneous setting for two subpopulation (weak and strong) was also considered in (Cha, 2016). Note that our approach here is applied to continuous frailty case. Moreover, our reasoning, in contrast to (Cha, 2016), is based directly on the numbers of minimal repairs observed in relevant time intervals and on the subsequent comparison of stages.

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