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Parameter Estimation In Bivariate Wiener Degradation Process Subject To Imperfect Preventive Maintenance Considering Different Observation Schemes

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In recent years, research on system deterioration modelling based on stochastic processes has predominantly focused on univariate processes. In practice, industrial systems usually have more complex structures with different interrelated components. For these systems, a multivariate degradation model is needed to describe the degradation evolution (Mercier et al., 2012).

In this work, a bivariate degradation model is studied (Lai, 1995). This bivariate degradation model describes the degradation evolution in a dependent two-component system. The dependence between components is described by using the trivariate reduction method (Bautista et al., 2023). Let $\{W_0(t), t \geq 0\}, \{W_1(t),\ t \geq 0\}$ and $\{W_2(t),\ t \geq 0\}$ be three independent univariate Wiener processes given by

$$W_1(t) = \mu_1 t + \sigma_1 B_1(t),\ W_2(t) = \mu_2 t + \sigma_2 B_2(t),\ W_0(t) = \sigma_0 B_0(t), \tag{1}$$

where B_0 and B_1 and B_2 are three independent standard Brownian processes. Starting from these independent Wiener processes, the degradation of each component at time *t* is modelled as follows.

$$X_1(t) = \mu_1 t + \sigma_1 B_1(t) + \sigma_0 B_0(t),\quad X_2(t) = \mu_2 t + \sigma_2 B_2(t) + \sigma_0 B_0(t). \tag{2}$$

Parameters μ_1 and μ_2 correspond to the drift parameters of X_1 and X_2 . σ_0, σ_1 and σ_2 stand for the variance parameters. We get that $X_i(t)$ is normally distributed with expectation $\mu_i t$ and variance $(\sigma_0^2 + \sigma_i^2)t$ for $i = 1,2$. The term $\sigma_0 B_0(t)$ introduces the dependence among the processes $X_1(t)$ and $X_2(t)$. Covariance between $X_1(t)$ and $X_2(t)$ is given by $\sigma_0^2 t$. The Pearson correlation coefficient between $X_1(t)$ and $X_2(t)$, θ, is invariant in time and equals to

$$\theta = \sigma_0^2 / (\sqrt{(\sigma_0^2 + \sigma_1^2)(\sigma_0^2 + \sigma_2^2)}\,. \tag{3}$$

To mitigate the degradation effect on the components, imperfect preventive maintenance actions are performed each T time units (with $T > 0$). These preventive maintenance actions (Mercier et al., 2019) reduce the degradation of each component in a ρ% with $0 \leq \rho \leq 1$. Let $Y_i(t)$ be the degradation level of the maintained component i at time *t*. The jump in the degradation due to the *n*-th maintenance preventive action on the component *i* is given by

$$Z_i(nT) = Y_i(nT+) - Y_i(nT) = (1-\rho)Y_i(nT) - Y_i(nT) = -\rho\, Y_i(nT). \tag{4}$$

Under this maintenance scheme, the degradation level of the maintained component i ($i = 1,2$) at time t is given by

$$Y_i(t) = \sum_{j=1}^{n}(1-\rho)^{n-j+1}(X_i(jT) - X_i((j-1)T)) + \left(X_i(t) - X_i(nT)\right), \quad nT \le t < (n+1)T. \tag{5}$$

It is easy to check that $Y_i(t)$ is normal distributed. Furthermore, the Pearson correlation coefficient between $Y_1(t)$ and $Y_2(t)$ is invariant in time and equal to $\theta = \sigma_0^2/(\sqrt{(\sigma_0^2 + \sigma_1^2)(\sigma_0^2 + \sigma_2^2)}$. On the other hand, the jumps $Z_i(nT)$ are also normally distributed and correlated $E[Z_i(nT)Z_i((n+1)T)] \neq E[Z_i(nT)]\, E[Z_i((n+1)T)]$.

We assume that the system is observed, and degradation data of each maintained component are obtained. The purpose of this work is to estimate the parameters of the bivariate Wiener model under the different observation schemes by using the degradation data. That is, the goal is the estimation of $\mu_1, \mu_2, \sigma_0, \sigma_1, \sigma_2, \rho$ and θ given the degradation data of the two maintained components.

The estimation of parameters $\mu_1, \mu_2, \sigma_0, \sigma_1, \sigma_2, \rho$ and θ depends on the observation schemes in which the data were obtained. As in (Leroy et al., 2023), two kinds of observations are obtained, the degradation increments between maintenance times and the observed jumps around maintenance times. In this work, the observation schemes are the following.

- Observation Scheme 1. Degradation levels of the maintained components are obtained between maintenance actions. Around the maintenance times, only the size of the jump in the degradation of each component due to the preventive maintenance actions is recorded.
- Observation Scheme 2. Degradation levels of the maintained components are obtained between maintenance actions. Around the maintenance times, the degradation levels of both components are observed just before and just after each maintenance action.
- Observation Scheme 3. Degradation levels of the maintained components are obtained between maintenance actions. Around the maintenance times, the degradation levels of both components are observed just after each maintenance action but not just before.
- Observation Scheme 4. Degradation levels of the maintained components are obtained between maintenance actions. Around the maintenance times, the degradation levels of both components are observed just before each maintenance action but not just after.
- Observation Scheme 5. Degradation levels of the maintained components are obtained between maintenance actions. Around the maintenance times, the degradation levels of both components are not observed neither just before nor just after each maintenance action.

To estimate the parameters $\mu_1, \mu_2, \sigma_0, \sigma_1, \sigma_2, \rho$ and θ, the likelihood function $L(\mu_1, \mu_2, \sigma_0, \sigma_1, \sigma_2, \rho, \theta)$ is derived. This likelihood function has two parts: the part linked to the degradation increments and the part linked to the jumps due to the maintenance actions. The part linked to degradation increments corresponds to the product of normal densities and it is common to the five observation schemes. The part linked to the jumps is more complex since the jumps are correlated. Furthermore, each observation scheme leads to a different likelihood for the jumps.

Under this framework, the maximum likelihood function is derived for each observation scheme. Numerical simulations are used to obtain the estimators of the parameters by using the maximum likelihood function. As particular case, the estimation of the parameters in the univariate case is also derived.

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