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# Quantum Reliability And Its Application In Quantum Networks

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Reliability is a crucial factor in system engineering, representing the ability of equipment to operate flawlessly within a specified timeframe. The concept of reliability gained significance alongside the emergence of quantum mechanics in the early 20th century. As industrial production expanded and quantum physics advanced independently, leading to groundbreaking technologies like nuclear energy, lasers, and semiconductors, the theory of reliability took its initial significant steps.

Fast forward to today, with the advancement of quantum technology such as quantum computing and quantum information, the promotion of quantum devices enhanced by quantum coherence has become a subject of intense fascination. Their immense potential in diverse fields such as computing, communication, sensing, and security has captured widespread attention. Consequently, the preservation of a quantum device's long-term functionality has emerged as a great concern, inevitably leading to the convergence of quantum physics and reliability engineering. Due to the existence of quantum properties, the reliability of systems needs to be re-evaluated. In a recent letter published by the author (Cui, Du and Sun, 2023), a reliability analysis framework based on quantum mechanics has been introduced.

Here, we present a reliability-assessment model of quantum network as an illustration. First consider a quantum communication channel between two node, Alice and Bob. They can send quantum states (a kind of message) by quantum teleportation (Bennett et al., 1993) using the entanglement pairs (also called EPR pairs) as the communication "channels". However, due to the noise, the channel's reliability will lose, which is governed by quantum dynamics. To protect "channels", one can implement some local quantum operations called purification (Bennett et al., 1996), which is a redundancy method based on quantum mechanics. Purification can be done multiple times to reduce the reliability-loss of the channel. The probability to successful generate an EPR pair with one round purification can be written as

$$r(t) = \frac{2e^{2\kappa t} + e^{4\kappa t} + 5}{4e^{4\kappa t} + 4}, \tag{1}$$

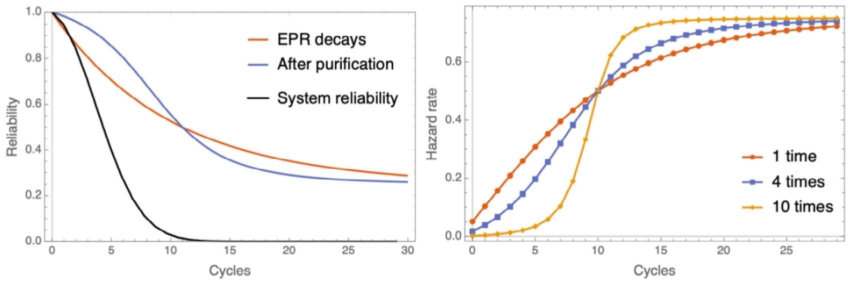
where $\kappa$ is the decreasing rate of EPR pairs, see the letter (Cui, Du and Sun, 2023) for detail derivations. Assume that Alice and Bob will communicate every time step, the time interval is $\Delta t$. The reliability $R(k)$ of this quantum communication channel, i.e. the probability of successfully communication between Alice and Bob for $k$ times, can be get through

$$R(k) = \prod_{i=0}^{k} r(i\Delta t). \tag{2}$$

The hazard rate of this quantum channel is

$$x(k) = -\frac{\ln R(k+1) - \ln R(k)}{\Delta t} = -\frac{\ln r((k+1)\Delta t)}{\Delta t}. \tag{3}$$

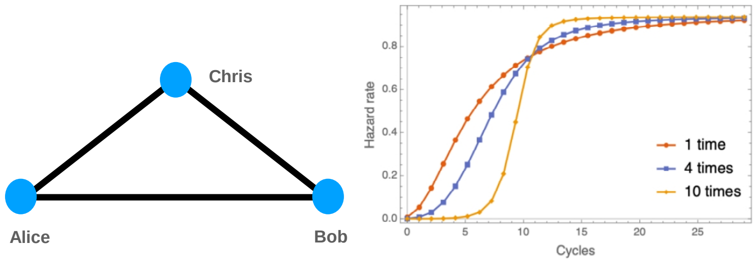
The results are depicted in Fig. 1, with Fig. 1(A) illustrating reliability and Fig. 1(B) representing hazard rate.




(A). (B).

Fig. 1. (A) The red line represents the decay of survival probability for EPR pairs (quantum communication channels), while the blue line depicts the survival probability of EPR pair after four rounds of purification. The black line corresponds to the system reliability calculated using Equation (2); (B) The hazard rate of the quantum communication channel with different purification times.

Next, consider a small network with three nodes, as illustrated in Fig. 2 (A). The reliability of the network here is set to be the connection of each two nodes. Regarding each edge as a component, the system become a 2-out-of-3 system. The reliability of the system can be calculated through $R_{\text{sys}} = R^3 + 3R^2(1 - R)$. Each edge is a quantum communication channel as described above, and the system hazard rate is illustrated in Fig. 2(B).




(A). (B).

Fig. 2. (A) A network with three nodes; (B) The hazard rate of the quantum network with different purification times.

Illustrated by the quantum network, we show how quantum reliability theory can analyze the reliability loss governed by quantum dynamics instead of classical stochastic process. Quantum reliability establishing a solid groundwork for analyzing the reliability of quantum devices. In future quantum engineering applications, quantum reliability will serve as a vital measure, enabling the evaluation and optimization of quantum devices' design. As this captivating journey unfolds, the alliance between quantum reliability and cutting-edge quantum technologies will shape a future of unparalleled reliability and innovation.

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