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# Data-Driven Stochastic Damage Identification With Conditional Invertible Neural Network (cINN)-Based Multilevel Bayesian Model Updating

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Structural Health Monitoring (SHM) has emerged as a pivotal technique for assessing damage in engineering structures, ensuring their safety, reliability, and serviceability. This technique is integral to identifying potential failures early, facilitating timely repair and maintenance actions. At the heart of SHM lies the challenge of damage identification, for which numerous methods have been proposed. Among these, the stochastic model updating-based approach stands out. It is celebrated for its capacity to bridge the gap between numerical simulations and experimental data, effectively handling the inherent uncertainties present in both (Farrar and Worden, 2007).

This approach hinges on comparing the distributions of updated model parameters between intact and damaged models. Such comparisons are instrumental in damage identification, risk assessment, and failure prognostics. Central to solving the inverse problem in stochastic model updating is Bayesian inference. This method leverages prior knowledge (prior distribution) and observational data to obtain a posterior distribution of uncertain model parameters, thereby maximizing the likelihood function. The likelihood function quantifies the probability of observing the data for different parameter values, making it a cornerstone of this approach (Lam, Yang and Au, 2018).

However, establishing the likelihood function poses significant challenges. It is often computationally demanding, owing to the need for high-dimensional integral calculations and the quantification of uncertainties. Additionally, the complexity of models can render analytical solutions infeasible (Bi et al., 2023). To address these issues, strategies such as approximate Bayesian computation (ABC) have been developed, including the use of a Bhattacharyya distance-based approximate likelihood function. Sampling methods like Markov Chain Monte Carlo (MCMC) and Transitional Markov Chain Monte Carlo (TMCMC) also play a crucial role. They allow for sampling from posterior distributions without directly computing the normalizing factor, a process integral to Bayes' Theorem (Bi, Broggi and Beer, 2019). Despite these advancements, challenges remain. There is an over-reliance on the precision of sampling methods, and the time-intensive nature of handling high-dimensional problems poses significant hurdles. These challenges can impede achieving the timeliness and accuracy required for effective SHM (Lye, Cicirello and Patelli, 2021).

In response to the aforementioned challenges, this work integrates the BayesFlow framework, as developed by (Radev et al., 2022), offering a novel approach to circumvent the computational and analytical hurdles in SHM. BayesFlow stands out by enabling the simulation-based training of cutting-edge neural network architectures, including deep neural networks and normalizing flows, to facilitate amortized posterior inference effectively. The essence of BayesFlow's architecture, depicted in Figure 1 (a), is encapsulated in two primary neural networks: the summary network and the inference network.

The summary network plays a critical role in condensing a variable-sized set of observations into a fixed-size vector of learned summary statistics. This process is crucial for handling the diverse and complex data typically encountered in SHM. On the other hand, the inference network focuses on discerning the true posterior distribution of model parameters based on these summary statistics. This is achieved by implementing the network as a conditional invertible neural network, featuring multiple coupling layers designed to accurately predict the parameter distributions from observational data post-training.

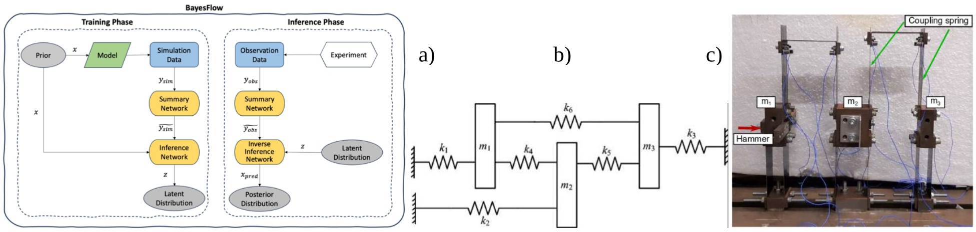


Fig. 1. (a) Architecture of BayesFlow; (b) 3-DOF spring-mass system; (c) Experimental rig case study.

A key aspect of BayesFlow's methodology is the manual selection of the summary network by researchers. This allows for the autonomous learning of the most informative statistics from the raw data. Meanwhile, the inference network's execution through conditional invertible neural networks ensures precise modeling of the true distribution of model parameters. When trained jointly, these networks promise rapid and precise predictions, significantly enhancing the efficiency and reliability of damage identification in SHM systems (Zeng, Todd and Hu, 2023).

This study harnesses the BayesFlow framework for stochastic model updating, applying it to the intricate task of damage identification. To evaluate the efficacy of the conditional Invertible Neural Network (cINN)-based method for stochastic damage detection, two distinct models were employed: a theoretical 3-DOF (degrees of freedom) spring-mass model, illustrated in Figure 1 (b), and a physical 3-DOF experimental rig (Isnardi, et al., 2023), depicted in Figure 1 (c), each subjected to various damage scenarios.

During the training phase, data is synthesized from a specified prior distribution and the simulation outputs of the model. This process ensures that BayesFlow is adeptly trained to perform inverse predictions, using observational data sequences to ascertain the true parameter distribution responsible for the observed damage. Damage scenarios are meticulously crafted to simulate varying degrees of stiffness reduction at specific locations within the models. This approach allows for a nuanced comparison of the posterior distributions of structural stiffness parameters in both damaged and undamaged states, facilitating not only damage identification but also localization and assessment. This is further enriched by calculating the probability of damage (POD), offering a quantitative measure of damage severity.

Moreover, this study advances the application of multilevel Bayesian inference. Unlike traditional methods that directly update structural parameters, this approach focuses on the hyper-parameters of these structural entities. Consequently, the outcomes of model updating are expressed through the marginal posterior densities of the structural parameters, with particular interest in their means and standard deviations. This nuanced approach allows for a more refined understanding of the structural health, encapsulating the uncertainties inherent in both the model and observed data.

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