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Exploring Initialization Strategies For Quantum Optimization Algorithms To Solve Redundancy Allocation Problem

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In the industrial context, system reliability is crucial to ensure efficient and safe operations, and redundancy allocation has been used for its maximization. Then, the Redundancy Allocation Problem (RAP) stands out, aiming to find the optimal configuration of redundant resources to optimize the reliability of a system subject to budget constraints (Araújo et al., 2022; Ouyang et al., 2019; Ramirez-Marquez; Coit, 2004).

In the literature addressing RAP, robust techniques are sought to deal with the increasing complexity of industrial systems, such as metaheuristics (e.g., Genetic Algorithm and Ant Colony). Concurrently, for different combinatorial optimization problems, methods based on quantum principles have been explored. Quantum Computing emerges as a potential approach, using properties like superposition and entanglement in its algorithms (Araújo et al, 2022). Among these quantum optimization algorithms, we can highlight the Variational Quantum Eigensolver (VQE) and the Quantum Approximate Optimization Algorithm (QAOA). However, the effectiveness of these algorithms is significantly influenced by the initial point choice, which impacts the convergence and quality of the solutions found (Xie et al., 2023; Moussa et al., 2022).

Therefore, this study aims to explore the impacts of different parameter initialization strategies on the results obtained by QAOA and VQE to solve RAP. For this purpose, methods such as random initialization, Latin Hypercube Sampling (LHS), clustering, and fixed point will be tested.

RAP formulation

The original formulation of reliability problems typically presents an objective function with nonlinear terms due to the reliability relationships among the components involved. To be integrated into quantum machines, this formulation needs to undergo a linearization process, as quantum machines use the Quadratic Unconstrained Binary Optimization (QUBO) as a model to map the systems (Glover et al., 2019). For linearization purposes, it is necessary to change the original formulation of maximizing system reliability to minimize the system failure probability. Therefore, the objective function is adjusted to the product of the components' quantity and the logarithm of the components' simultaneous failure probability. The formulation for a parallel-active system is presented in (1)-(4). The variables x_k indicate the quantity of components of type k, $k = 1, \dots, ct$, and the objective function considers their cost (c_k) and reliability (R_k). Thus, the goal is to minimize the objective function subject to cost constraints (C), and limits on the total number of allocations (n_{min} e n_{max}).

$$Min \sum_{k=1}^{ct} x_k \cdot ln\,(1 - R_k) \tag{1}$$

$$s.t. \sum_{k=1}^{ct} x_k \cdot c_k \leq C \tag{2}$$

$$n_{min} \leq \sum_{k=1}^{ct} x_k \leq n_{max} \tag{3}$$

$$x_k \in \{0, 1, \dots, n_{max}\} \tag{4}$$

Materials, Methods, Results

In this study, the quantum algorithms QAOA and VQE were implemented using the Qiskit library and the Python programming language. During the experimental phase, result reproducibility was ensured by maintaining consistency using a global random seed set to 10,598. The quantum algorithms were executed on a quantum instance that configured the simulation environment, utilizing the BasicAer library as the backend, through the 'qasm simulator' that provides 24 quantum bits (qubits).

In these configurations, four initialization strategies were explored for the key parameters of the algorithms, including random, LHS, clustering, and fixed point. Performance analysis was conducted over 30 iterations for each configuration, aiming to provide a robust understanding of the algorithms' behavior with each initialization.

Additionally, two main metrics were used to evaluate the performance of the initialization strategies: the algorithm's convergence to the optimal solution and the runtime. Convergence refers to the algorithm's ability to reach the optimal solution over iterations, while runtime measures the temporal efficiency of applying these strategies.

The conducted experiments demonstrated the sensitivity of the quantum algorithms, QAOA and VQE, to initialization strategies for the following instance: $ct = 3$; $n_{max} = 2$; $n_{min} = 1$; $R_1, R_2, R_3 = 0.6086, 0.9860, 0.6999$; $c_1, c_2, c_3 = 3, 6, 5$; $C = 6$. The variation in convergence rates toward the optimal solution, when exploring different initial points, highlighted the significant influence of the choice of initialization strategy on the overall performance of the algorithms, as shown in Table 1.

Table 1. Results of applying QAOA and VQE to instance 1.

Initialization	QAOA		VQE	
	Percentage of convergence to the optimal solution	Average execution time (seconds)	Percentage of convergence to the optimal solution	Average execution time (seconds)
Random	10.00%	39.51	6.67%	117.56
LHS	3.33%	49.06	16.67%	129.22
Clustering	3.33%	69.41	16.67%	110.63
Fixed point	0.00%	32.36	23.33%	82.09

VQE consistently provided better results concerning convergence for the optimal solution when compared to QAOA. The fixed-point initialization strategy demonstrated faster execution times, highlighting its temporal efficiency. However, it showed a limitation in convergence to the optimal solution, especially in the QAOA. Furthermore, the Random method had better results than LHS and clustering for QAOA, but the situation reversed for VQE. LHS and clustering exhibited similar performances in terms of convergence for each algorithm. The direct comparison between QAOA and VQE, regarding convergence and execution time, emphasized important trade-offs. While VQE demonstrated more consistent convergence to the optimal solution, regardless of the initialization method, QAOA excelled in temporal efficiency.

RAP stands out as a support for maximizing the reliability of systems. The results obtained in this study emphasize the relevance of choosing the initial parameters for the quantum algorithms QAOA and VQE so that these methods can reach optimal solutions. Thus, by exploring different initialization strategies, significant impacts on the convergence and runtime of the algorithms were observed.

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