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Incorporation Of Non-Gaussian Shell Imperfections By Nataf Transformation In Efficient Random Field Simulation

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In civil and aerospace engineering, shells are highly demanded structures. Their curved shape is the reason that they can carry loads very efficiently. This enables a high load-bearing capacity with low dead weight and a thin-walled design. However, the dominant failure mode of thin-walled shells is buckling, which is significantly influenced by material and geometric imperfections. Even minor variations in these imperfections can have substantial impact on the load-bearing capacity. The shape of imperfections is frequently uncertain and, in some cases, only limited measurements are accessible. Traditional design concepts are based on very conservative design factors, also known as knockdown factors (KDFs). The development of reliable and more economical design concepts is therefore still part of numerous research projects, see for example (Wagner et al., 2020).

One possible approach is to model spatially varying imperfections by employing random fields and autocorrelation functions, see for example (Broggi and Schuëller, 2015; Fina et al., 2023; Lauterbach et al., 2018). Therefore, a range of assumptions concerning the random field has to be established, such as determining the correlation lengths and autocorrelation functions to be used and whether the field is homogeneous or non-homogeneous. In (Fina et al., 2021) polymorphic (mixed/hybrid) uncertainty models are defined to model random shell imperfections based on limited data. However, due to traditional engineering approaches, it is often assumed that geometric random imperfections are Gaussian distributed and the influence of possible non-Gaussian distributions is neglected. Only a few studies are conducted on the simulation of non-Gaussian shell imperfections. One example is the work by (Papadopoulos et al., 2009), where stochastic non-Gaussian material and thickness properties are considered.

The presented work focuses on geometric imperfections modeled as geometric deviation in radial direction of a cylindrical shell. A detailed analysis is carried out to explore how non-Gaussian geometric imperfections influence the buckling behavior. Therefore, a data base for geometric imperfections from (Arbocz and Abramovich, 1979) is analyzed. The data base contains shell types with different sizes and manufacturing processes. The measured geometric imperfections are reproduced as Fourier series with the given Fourier coefficients. In (Fina et al., 2019, 2020) an approach is presented to evaluate correlation parameters for a simulation of the imperfections as random fields based on given Fourier series.

Samples of unrolled geometric imperfections of a cylindrical shell from (Arbocz and Abramovich, 1979) are depicted in Figure 1. The geometric deviations are analyzed across the samples. It is determined that for some imperfection fields the marginal distributions of the deviations are non-Gaussian distributed.

In case of non-Gaussian marginal distributions, the Nataf transformation is required to simulate correlated random variates (Liu and Der Kiureghian, 1986). The simulation of non-Gaussian processes or fields with the corresponding non-Gaussian covariance matrix is based on transformations into the Gaussian space (Grigoriu, 1998; Vořechovský, 2008).

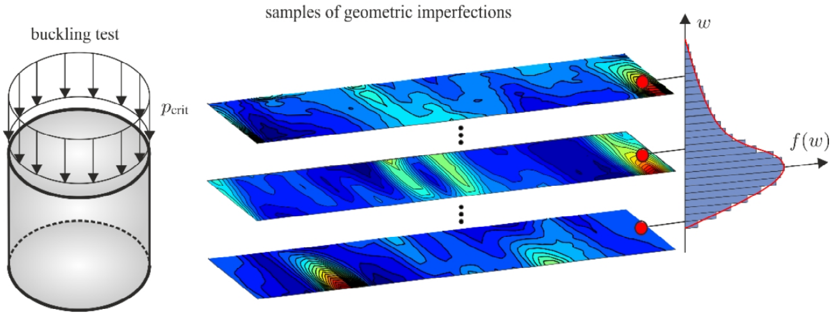

Fig. 1. Analysis of the distribution function of measured geometric imperfections.

The basic idea of the Nataf transformation is to calculate equivalent correlation coefficients in the standard normal space to obtain correlated realizations of non-Gaussian random variables (Xiao, 2014). For the presented application, first of all empirical distribution functions of the geometric deviations are evaluated based on the experimental data. The corresponding covariance matrix is generated in the standard normal space. Then, the covariance matrix is modified to generate realizations of non-Gaussian random fields using the Karhunen-Loève expansion. These non-Gaussian random fields are applied as geometric deviation in radial direction of nodes on a finite-element model of a cylindrical shell under axial pressure. Thus, Monte Carlo simulations are performed to calculate the output distribution functions of the buckling loads. For comparison, simulations are also carried out with Gaussian random imperfections.

The presented work contributes to a more realistic modeling of geometric imperfections with random fields. In particular, the distributions of the imperfection fields are analyzed based on experimental data. The main objective is to quantify the influence of non-Gaussian fields on the buckling behavior and to discuss whether the simplified traditional engineering assumption of Gaussian random imperfection fields is acceptable.

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