

Inspection And Replacement Policy For Degrading System With Imperfect Partial Maintenance Effects

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We are focused on the cost modelling and optimization of a maintenance policy applied to a new Wiener-based degradation model with partial maintenance effects. The considered maintenance policy involves inspection, imperfect maintenances, and as-good-as-new replacements. Its decision variables are the inter-inspection time and the preventive replacement threshold. This maintenance policy is optimized based on the average maintenance cost, which is determined using the Markov-renewal properties of the degradation process on the maintained system. It is also explored the impact of specific cost coefficients and model parameters on the optimal, maintenance policy.

Let consider a system whose degradation is represented by a Wiener-based degradation model with partial maintenance effects. The imperfect maintenance actions only impact a part of the system. This imperfect maintenance effect is described by an ARD_1 -type model (Mercier and Castro, 2019). More precisely, the degradation process is composed of two underlying processes: $X^U = \{X^U(t)\}_{t \geq 0}$ the unmaintained component process and $X^M = \{X^M(t)\}_{t \geq 0}$, the maintained component process. Between maintenance actions, degradation results from the sum of these two processes, with maintenance exclusively affecting the X^M component. The underlying processes, X^U and X^M , are Wiener processes with drift, defined by parameters including $\mu_U, \mu_M, \sigma_U^2, \sigma_M^2$ and r_{UM} that respectively stands for the drift parameters, the variance parameters and the correlation coefficient between the two processes. Let $Y(t)$ be the degradation level of the whole maintained system. Let $\tau_1, \tau_2, \dots, \tau_k$ denote the maintenance times, i.e. the scheduled times for maintenance actions. The parameter $\rho \in [0,1]$ represents the maintenance efficiency of the imperfect maintenance actions on the maintained component degradation process. For all $j \in \{1, \dots, k\}$, and all $t \in [\tau_{j-1}, \tau_j]$, the degradation level is defined as follows: $Y(t) = X^U(t) + X^M(t) - \rho X^M(\tau_{j-1})$. Moreover, the degradation level just before the j^{th} maintenance action, denoted as $Y(\tau_j^-)$, is given by: $Y(\tau_j^-) = X^U(\tau_j) + X^M(\tau_j) - \rho X^M(\tau_{j-1})$ (Leroy et al., 2024). When the degradation level $Y(t)$ reaches a failure threshold L , the system is considered as failed and unavailable until the next replacement.

The system degradation is periodically inspected (with period $\tilde{\tau}$) before every preventive maintenance actions. Inspections are immediately followed by either imperfect maintenance or preventive replacement. The inspected degradation level just before each maintenance, denoted as $Y(\tau_j^-)$ for all $j \in \{1, \dots, k\}$ is considered to decide between an imperfect maintenance action or an as-good-as-new replacement. Both preventive and replacement maintenance actions are assumed to be instantaneous. M represents the preventive degradation threshold and L is the corrective degradation threshold. The maintenance decision is governed by the following rules that apply for each interval $[\tau_{j-1}, \tau_j]$:

- If for all j , there exists $t \in [\tau_{j-1}, \tau_j]$ such that $Y(t) \geq L$, then a failure occurs at time t when degradation first exceeds the threshold L , leading to temporary unavailability. The system undergoes a corrective replacement at the next maintenance time, such that $Y(\tau_j^+) = 0$. If the degradation does not exceed L between two inspection times, either imperfect maintenance or preventive replacement is carried out.

- If $Y(\tau_j^-) < M$, an imperfect maintenance is performed. In that case, degradation level is reduced to a certain level proportional to the degradation accumulated on X^M since the last maintenance action. Thus, degradation right after the imperfect maintenance is defined as follows: $Y(\tau_j^+) = X^U(\tau_j) + (1 - \rho)X^M(\tau_j)$.
- If $M \leq Y(\tau_j^-) < L$, a preventive as-good-as new replacement is executed on the entire system, initiating a new life cycle, such as $Y(\tau_j^+) = 0$.

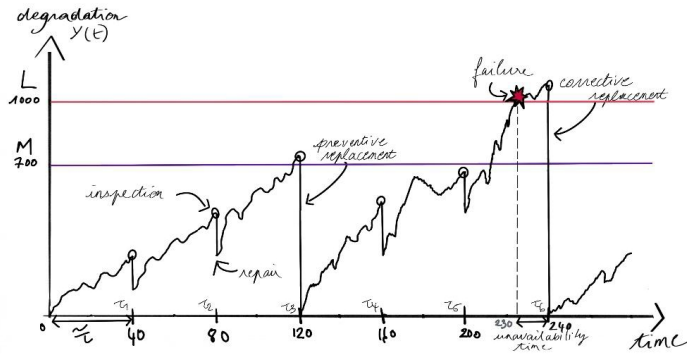


Fig. 1. System degradation over two life cycles.

The preventive replacement threshold M and the inspection period $\bar{\tau}$ are the decision variables for the considered maintenance policy. To suggest improved maintenance policies, optimal values of the decision variables must be assessed by minimizing the maintenance cost over time. The complexity of expressing analytically the maintenance cost due to imperfect preventive maintenance actions on the system can be bypassed by considering a regenerative Markov process. Specifically, after each replacement, degradation is reset to zero, and the following deterioration process is independent of prior events. Then, the renewal theory can be applied (Cinlar, 1969) and optimizing the asymptotic cost per time unit is equivalent to optimize the average cost over a system life cycle divided by the expectation of the length of a cycle. An alternative approach consists in optimizing the cost between two imperfect maintenances (Cocozza-Thivent, 2000; Grall et al. 2002; Corset et al., 2022). Thus, between two inspections, the model can be described as a semi-regenerative Markov process. The stationary measure π of the Markov chain defined by the degradation levels after imperfect maintenance actions on the continuous state space $[0, M]$ can be derived and used to evaluate the cost. Unlike the first purely simulation-based approach, the assessment of the asymptotic cost per time unit, when considered between two inspections, includes both analytic expressions of the expectation of the cost (under the stationary distribution) given the current degradation level and simulations of the post-maintenance degradation levels. Based on this second method, which is less time-consuming, the optimal values of the inter-inspection time $\bar{\tau}$ and the preventive threshold M are assessed by minimizing the asymptotic maintenance cost per time unit.

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