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Leveraging Two Reliability Approaches For Estimating Failure Probability And Its Sensitivity

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A common challenge when designing engineering systems is that not all parameters that affect performance are known in a crisp way. For example, parameters such as loading, material properties or deterioration process may be subject to significant levels of uncertainty. In such a case, probability models lend themselves to characterize the associated uncertainty. Thus, the performance of a system becomes uncertain as well and can be described, for example, in terms of the failure probability. This probability measures the chances that the system undergoes an undesirable behavior (Song et al., 2023). In addition, another means to characterize the effect of uncertainty is to quantify the sensitivity of the failure probability with respect to the distribution parameters associated with the underlying probabilistic model. Such sensitivity can be expressed in terms of the derivative of the failure probability with respect to, for example, the expected value of an uncertain parameter of the model. Such sensitivity is instrumental for pinpointing the most influential parameters of a given problem (Torii, 2020).

Calculation of the failure probability and its sensitivity is a problem which has received considerable attention in the past. Both quantities can be calculated resorting to different approaches, such as approximate reliability methods and simulation techniques. Within the first class of approaches, possibly the most popular approximate method is the so-called First Order Reliability Method. This method heavily relies on linearization and the concept of the design point and can provide closed-form, analytical expressions for evaluating both the probability and its sensitivity (Bjerager et al., 1989). The second class of approaches encompass the Monte Carlo method and its advanced variants and can calculate the probability and its sensitivity by generating samples of the uncertain parameters of the problem and evaluating the system's response for each of those realizations.

A comparison of the performance of approximate reliability methods and simulation techniques indicates that they usually offer a trade-off between precision, accuracy, and numerical costs. Approximate reliability methods can be quite efficient from a numerical viewpoint for certain classes of problems, as their implementation entails performing few deterministic analyses of the underlying system. Moreover, they produce precise results, as they are based on closed-form analytical expressions (Hohenbichler et al., 1986). However, their accuracy is unknown due to the hypothesis of linearization. Simulation techniques can offer accurate results, as they do not introduce assumptions about the behavior of the underlying system. However, to produce sufficiently precise estimates, a large number of evaluations of the system's performance is required for different realizations of the uncertain parameters (Rubinstein et al., 2016).

The preceding discussion highlights the advantages and disadvantages of approximate reliability methods and simulation techniques. In such a scenario, this contribution investigates whether it is possible to leverage on approximate reliability methods to improve the results obtained with a simulation technique. More specifically, the focus is on investigating whether the performance of Importance Sampling (IS) using Design Points (Schuëller et al., 1987) together with the First-Order Reliability Method (FORM) can lead to improved estimates of the failure probability and its sensitivity with respect to distribution parameters. In other words, the aim is to leverage on the closed-form results of FORM to improve the estimates obtained with IS. Although this is a topic that has been already studied in the past for failure probability estimation (Fujita et al., 1988), the aim is to shed new light on this issue by resorting to the framework provided by Control Variates with Splitting (Avramidis et al., 1993). In a nutshell, Control Variates provides the means for exploiting correlations between two or more estimators to produce improved estimators with minimal variance (and hence, a high level of precision). The Splitting approach is helpful for setting the optimal control parameters associated with the implementation of Control Variates and consists of generating estimators of the sought quantities based on subsets of realizations. Therefore, the proposed framework benefits from the precision associated with the estimates produced by FORM and the accuracy of the estimates associated with IS. The application of the proposed framework is illustrated by means of a practical example.

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