Advances in Reliability, Safety and Security, Part 1 – Kołowrocki, Dąbrowska (eds.)
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Advances in Reliability, Safety and Security
ESREL 2024
Monograph Book Series

Bayesian Networks In Safety And Reliability

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Abstract

Bayesian Networks (BNs) have emerged as a widely embraced tool for high-dimensional probabilistic models. Their appeal lies in the intuitive graphical representation, which effectively captures engineers' understanding of complex systems. These models excel in delineating the subtleties of complex and dynamic realities, thanks to their intuitive visual representation. Supported by robust theoretical underpinnings and streamlined, user-friendly computational implementations, BNs offer impressive capabilities. Their popularity stems from their adaptability to data-rich applications and seamless integration of expert input. Among the others, BNs have found extensive applications in safety and reliability analyses. This paper showcases instances of BN applications in safety analysis, directs readers to authoritative overview articles on their applications in reliability, and outlines the latest advancements in copula-based BNs.

Keywords: reasoning under uncertainty, graphical models, Bayesian networks, copula.

1. Bayesian Networks (BNs)

The qualitative part of a BN is represented by a directed acyclic graph (DAG), G = (V, E), with nodes $V=\{v_1,\dots,v_d\}$ and directed edges (arcs) E. Any sequence of arcs $v_{i_1}\to\cdots\to v_{i_k}$, is called a ***path*** from node v_{i_1} to node v_{i_k} in G. The graph G is called a ***acyclic*** if it does not contain a path that starts and ends at the same node. For each edge $v\to w$ in E, the node *v* is said to be the ***parent*** of *w* and *w* is said to be ***child*** of *v*. Moreover, we denote as ***pa(v) (ch(v))*** the set of ***parents (children)*** of *v*. Since G is a directed acyclic graph, its nodes can be ordered such that parents appear earlier in the order then the children. A simple DAG with three nodes is shown in Fig. 1b. Node *T* has two parents *X* and *Y*, hence $pa(T)=\{X,Y\}$. *T* has no children, hence $ch(T)$ is empty.

The nodes in V correspond to random variables $X_1,\dots,X_d$. We assume that if two nodes are connected by an arc then the corresponding variables are directly related; and if there is no arc between two nodes v_i and v_j and v_i is earlier in the ordering of nodes than v_j, then corresponding variables X_i and X_j are conditionally independent given variables corresponding to all parents of node v_j (Pearl, 1988; Lauritzen,1996; Koller and Friedman, 2009). The quantitative part of BN models is composed of the conditional distributions of each node given its parents. Then the joint distribution of $(X_1,\dots,X_d)$ is equal to the product of these conditional distributions.

$$P_V(x_V)=\prod_{v\in V}P_{v|pa(v)}(x_v|x_{pa(v)}). \quad (1)$$

If all random variables $X_1,\dots,X_d$ are discrete then the product of conditional probability tables on the right hand side of (1) gives us the joint probability mass function of $X_1,\dots,X_d$. These BNs are called *discrete BNs*.

Bayesian Networks (BNs) serve as versatile generalizations of various models that have proven effective in the domains of safety and reliability. For instance, they can encompass fault tree (FT) models which excel at modeling complex engineering structures and are renowned for their proficiency in depicting failure occurrences within such systems. However, it is evident that because FTs rely on basic, intermediate, and top events with only two states, representing binary functions, they exhibit limitations when it comes to describing factors like human contributions to failures in engineering systems. BNs offer the possibility of modeling discrete variables with many states or continuous variables with versatile dependence structures. Even though BN's representation of FTs may be less visually intuitive and entail greater computational and memory resources, they can accommodate more flexible relationships between these variables beyond "AND" and "OR" gates. In Figure 1 a simple FT model and its BN representation is shown.

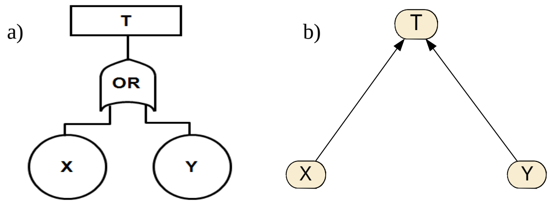

Fig. 1. (a) FT; (b) BN graph structure, corresponding to FT.

Bayesian Networks have gained widespread popularity as a powerful tool for conducting reasoning under uncertainty, due to their ability to be conditioned on available evidence. Consequently, in the model presented in Figure 1, one can explore the likelihood of event T occurring given that X equals 1 (indicating a failure), or observe how the probabilities of failure of both X and Y shift in case of the event of T's failure.

BNs can model the relationships between various components in a system and their failure modes as well as the dependency between repair times. This is useful in diagnosing faults and predicting future system behaviour. They can be used to model dependencies between different components in a supply chain, including suppliers, transportation modes, and warehouses, hence can help in optimizing the reliability of the supply chain. BNs are particularly suitable for modelling complex systems' reliability and availability.

BNs are mostly applied to model dependencies for discrete distributions due to the available powerful computational algorithms and numerous computer implementations. Extensive review can be found in (Weber et. al. 2010) and (Cai et al., 2019) where additional references are included. Moreover, discrete BNs have been applied in (Wooff et al., 2018) to software testing. BNs have been successfully extended and widely applied for the dynamic case. To represent continuous data, the following approaches are adopted: direct models (Gaussian and conditional Gaussian), models based on discretization and copula-based models. In this paper we concentrate on the discrete and the copula-based BNs.

1.1. Discrete BN in occupation safety

In this section, we present an example of a discrete Bayesian Network (BN) model developed as part of a project commissioned by the Ministry of Social Affairs and Employment in the Netherlands. The aim of this project was to assess and potentially mitigate risks for workers. Originally, the model representing a ladder accident was depicted using a 'bowtie' diagram. The central event, a fall from a placed ladder (F), is defined as a fall resulting in either fatal or serious physical and/or mental injury, leading to hospitalization or observation within 24 hours, along with suspicion of permanent physical or mental impairment. This fall can be attributed to the failure of one of the primary safety barriers (PSBs): Ladder Strength (SR), Ladder Stability (SL), and User Stability (SU). It is assumed that the failure of any one PSB is sufficient to lead to the fall. These PSBs are influenced by support safety barriers (SSBs): Placement and Protection (PP), Right Ladder (RL), and Ability (AB). The SSBs can be influenced by Management.

On the right-hand side of the Placement Ladder BN, there are three nodes influencing the consequences (C) of the fall, namely, Height (H), Medical attention (M), and Age of the victim (A). The majority of nodes in this BN have discrete states, with only the 'Age' of the victim and the 'Height' of the fall that can be modeled as continuous variables. However, in the project, they were discretized due to insufficient data as shown in Figure 2 (implemented in Netica software). The model's assumptions, along with the corresponding data, were adequate for quantifying this discrete BN. These included:

- the probability of support safety barriers given the loss of one of the primary safety barriers and subsequent fall;
- the probability of the loss of one of the primary safety barriers given a fall;
- the probability of a fall;
- the unconditional probability of various combinations of support safety barriers.

The first two probabilities, as well as the number of accidents required to determine the probability of a fall, were derived from available data. However, the exposure necessary to calculate the probability of a fall and the unconditional probability of support barriers had to be obtained through structured expert judgment (Cooke, 1991), as detailed in (Kurowicka et al., 2008). The quantification of the right-hand side of the model was based on empirical data.

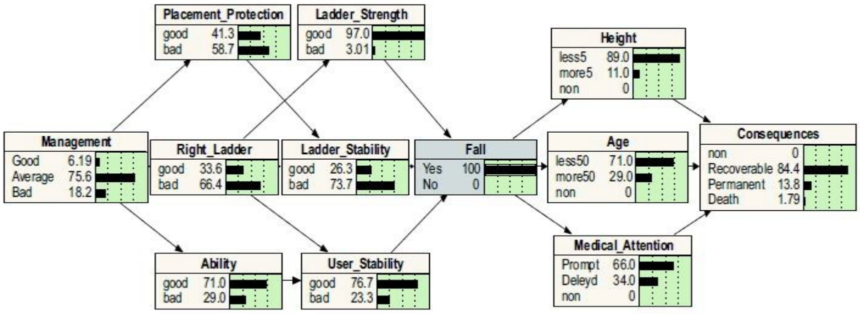

Fig. 2. Placement Ladder BN conditionalized on the event of fall.

This model enables us to explore, for instance, the impact of different management types on the likelihood of a fall from a ladder, as well as to forecast the resulting consequences. Similar models have been developed within this project to depict various other types of occupational accidents.

1.2. Gaussian and linear Gaussian BNs

In case $X_1, \ldots, X_d$ are absolutely continuous then their joint density in (1) is the product of the conditional densities and different types of such densities can in principle be chosen. The most popular are *Gaussian BNs*, where the conditional density of each node is Gaussian with means that are linear functions of parents and constant variances. Such representation of BNs is equivalent with the specification of the joint Gaussian distribution of variables corresponding to nodes of the DAG; see e.g. (Koler and Friedman, 2009).

Another extension of BN is so called Conditional Gaussian BNs, that allow discrete and continuous variables; see e.g. (Koler and Friedman, 2009). A recent application of linear Gaussian BNs to tank corrosion problem is presented in (Portinale, 2023) and to fault diagnosis in (Lou et al., 2020).

2. Copula-based BNs

The restrictions of Gaussian and Conditional Gaussian BNs can be relaxed by the application of copulas. A copula is a distribution on the unit hypercube with uniform marginal distributions (Joe, 2014). It contains all the information about the dependence between elements of a random vector. One can extract such a dependence structure corresponding to e.g. multivariate Gaussian distribution in the form of a Gaussian copula and use it to construct a BN with this copula and different marginal distributions. Other types of copulas are available that are able to model asymmetries and/or tail dependencies. In Figure 3 two bivariate copula densities are presented.

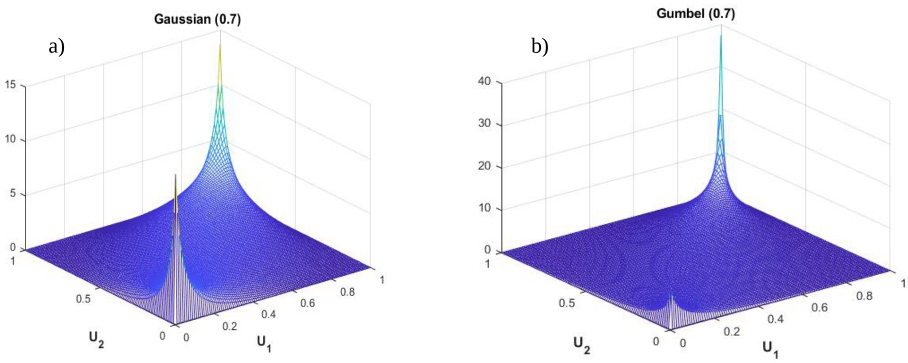

Fig. 3. Copula densities: Gaussian (a) and Gumbel (b) with the same strength of dependence measured with Spearman correlation equal to 0.7.

In (Kurowicka and Cooke, 2005), a copula-based BN approach was introduced. It is based on the representation of each conditional density in the factorization in (1) using a sequence of bivariate (conditional) copulas (see also (Bauer et al., 2012; Bauer and Czado, 2016)). For a node that has more than one parent, the convention is to order its parents (construct a parental ordering) and assign several copulas. First, a copula is assigned between the node and its first parent; then a copula is assigned between the node and its second parent conditional on the first parent; then a copula is assigned between the node and its third parent conditional on the first and the second parents. This process continues for all parents of each given node. Note that all these (conditional) copulas are unconstrained, in the sense that they can be chosen independently of each other and still form a valid density.

Definition 1

A ***pair-copula Bayesian network*** (PCBN) is composed of:

- (qualitative part) a DAG, G and a set of orders of parents of each node denoted as O,
- (quantitative part) marginal densities of variables corresponding to each node in G and the set of conditional copulas $c_{wv|pa(v;w)}$, where ***pa(v;w)*** denotes a set of parents of *v* before *w* in the order O (for nodes without parents $c_{wv|pa(v;w)} = c_{wv}$ by convention).

The joint density $P_V(x_V)$ in (1) in case of pair copula specification, can be rewritten

$$P_V(x_V) = \prod_{v\in V} P(x_v)\ \prod_{w\in pa(v)} c_{wv|pa(v;w)}\left(F_{w|pa(v;w)}\left(x_w \middle| x_{pa(v;w)}\right), F_{v|pa(v;w)}\left(x_v \middle| x_{pa(v;w)}\right)\right), \quad (2)$$

where the functions $F_{w|pa(v;w)}\left(x_w \middle| x_{pa(v;w)}\right)$ can be computed from copulas assigned to the arcs of BN. This distribution is composed of product of margins (the first product in (2)) and the product of copulas for each node and its parents (the second product in (2)). This second product is a factorization of the joint copula corresponding to the joint density, which can be denoted as c_V.

When all copulas in the factorization (2) are Gaussian copulas, then the joint distribution of variables corresponding to nodes in the graph is the distribution with copula c_V, which is the joint Gaussian copula. This makes computations in the model much more traceable and efficient. In the next subsection the application of gaussian copula BN is presented.

2.1. Gaussian copula-based BNs in air transport safety

The Netherlands Ministry of Transport, Public Works and Water Management commissioned a project on a Causal Model for Air Transport Safety, known as CATS. This model aimed to describe the gate-to-gate risks inherent in the complete aviation system. Aviation accidents result from a combination of various causal factors, including human errors, technical failures, and environmental and management influences, leading to specific accident categories like loss of control, collision, fire, etc. The causes and consequences of these accident categories vary depending on the phase of flight in which they occur, such as taxi, take-off, en-route, etc.

Event Sequences (33 incorporated in CATS) leading to accidents were developed and depicted in Event Sequence Diagrams (ESD), which were categorized according to flight phases: Taxi, Take-off, Climb, En-route, and Approach and Landing. For each pivotal event in the ESDs, Fault Trees (FTs) were developed and quantified using data obtained from ICAO's ADREP database, provided by airlines and airports. In over 100 instances in the final model, human intervention is required to prevent an accident. The probability that these actions do not result in the desired effect is described in human performance models (HPM) for the crew, ATC controllers, and maintenance technicians. These models contribute to the influence on human error probability included in FTs.

Figure 4 presents the HPM for the crew, which is a Gaussian copula BN. The marginal distributions of the nodes are quantified using existing data, except for nodes 6, 7, 9, and 10, whose distributions were assessed by experts following the Classical Model protocol (Cooke, 1991). The dependencies in these models (in form of Spearman (conditional) rank correlations) were obtained from experts using the procedure presented in (Morales et al., 2008), which is based on assessing exceedance probabilities. Similarly, two other HPM models were built and quantified. Through functional connections, all parts of the model were integrated into the CATS model presented in Figure 5. A comprehensive description of the model can be found in the final report for this project (Ale et al., 2008).

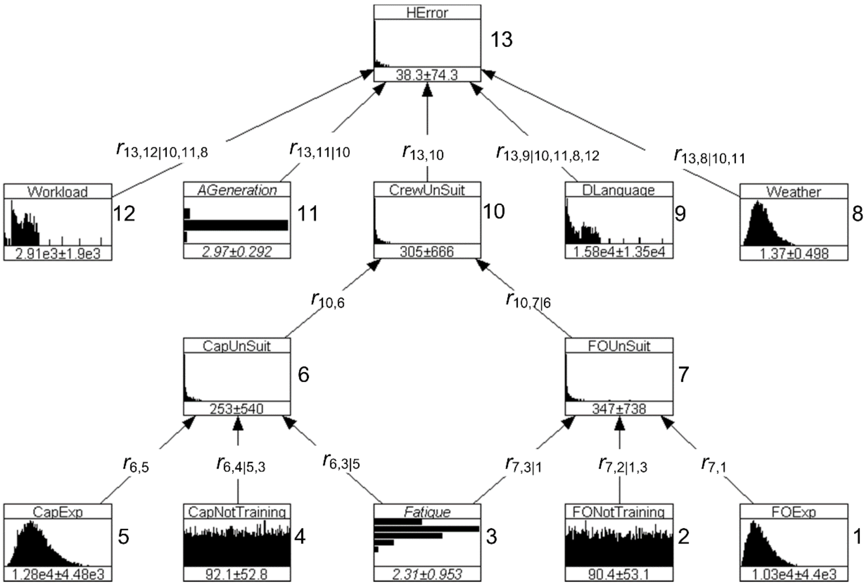

Fig. 4. HPM for the crew.

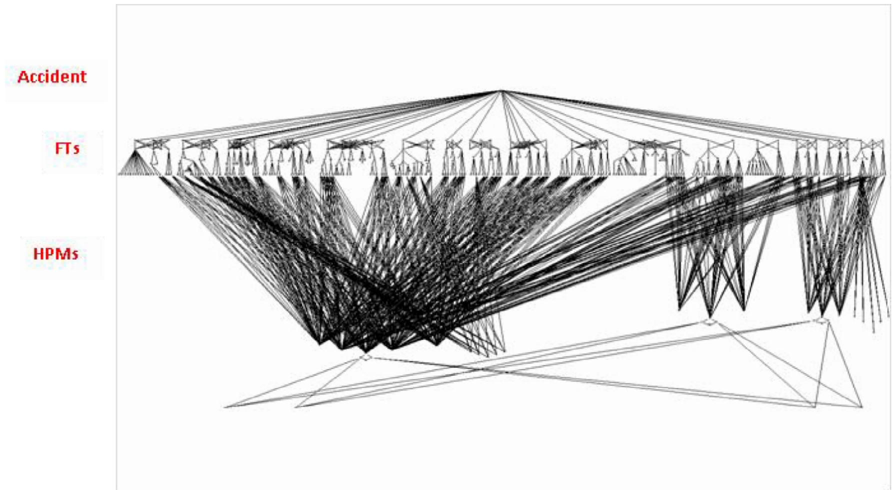

Fig. 5. CATS model with top node representing an Accident, middle part contains ESDs and FTs developed for these ESDs and underneath HPM models are placed.

In (Ale et al., 2010) CATS model was used to analyse the crash of Turkish Airlines TK 1951 flight approximately 1.5 km before the intended runway at Amsterdam Schiphol Airport in 2009. The information about parameters and causes the crash were mapped on the model. The results have shown that the probability of a crash predicted by the model increases dramatically if instruments command a trajectory towards terrain and the crew does not notice that (exactly what happened in TK 1951 accident). The analysis show that an integrated model can be used to detect vulnerabilities of the air transport system.

Other applications of Gaussian copula-based BN can be found e.g. in (Hanea et al., 2015) and (Delgado-Hernandez et al., 2012).

2.2. Pair-Copula BNs

The dependence realized by the Gaussian copula might not be sufficiently flexible for modeling complicated relationships that are asymmetric and/or tail dependent. If one wants to apply the representation presented in (2) in case the copulas assigned to each arc of the DAG are not necessarily Gaussian some computational issues can occur.

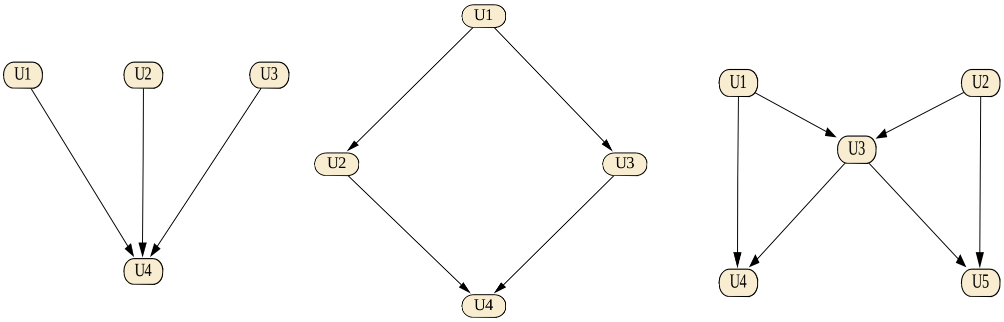

Fig. 6. (a) BN with v- structure; (b) BN diamond structure 4 nodes; (c) BN interfering v-structures.

Let us consider three graph structures in Figure 6. For the v-structure presented in Figure 6 (a) the following copulas can determine dependencies represented by arcs in this graph: c_{14}, $c_{24|1}$ and $c_{34|12}$ and the joint copula density is

$$c_V^a(u_1, u_2, u_3, u_4) = c_{14}(u_1, u_4)c_{24|1}(u_{2|1}, u_{4|1})c_{34|12}(u_{3|12}, u_{4|12}).$$

Note that in the formula above the order of parents for node 4 was chosen to be 1, 2, 3 and that the conditional margins $F_{3|12}(x_1|x_1\, x_2)$ are now simplified to $u_{3|12}$. If instead the order of parents of node 4 was 3, 1, 2 then we would require copula c_{34}, $c_{24|3}$ and $c_{14|23}$ and the joint copula density would be the product of these copulas. In this case, the arguments of the conditional copulas are easy to compute as all parents are mutually independent, hence $u_{2|1} = u_2$, $u_{3|12} = u_3$ and $u_{4|1}$ is computed from the specified copula c_{14} by differentiating the distribution function (cdf) of this copula with respect to u_1. Similarly, $u_{4|12}$ can be computed from cdf of copula $c_{34|12}$. Other parental orderings could be used as well and they would also not lead to computational issues. In general, when G is such that there is at most one path between each node (G is then called a *multitree*), then for each node, the variables corresponding to the parents of this node are independent and there will be no issue in computing the arguments of the (conditional) copulas in PCBN.

This is in contrast to the diamond structure presented in Figure 6 (b). In this case the copula density corresponding to the density, denoted now as f_V^b, is:

$$c_V^b(u_1, u_2, u_3, u_4) = c_{12}(u_1, u_2)c_{13}(u_1, u_3)c_{24}(u_2, u_4)c_{34|2}(u_{3|2}, u_{4|2}).$$

In this case the margin $u_{3|2}$ cannot be deduced directly and needs to be computed by integration:

$$u_{3|2} = \int_0^1 c_{12}(w_1, u_2) \int_0^{u_3} c_{13}(w_1, w_3) dw_3\, dw_1$$

The computational difficulties are caused by the undirected cycle with length larger than three, U1-U2-U4-U3-U1, in the undirected graph obtained from the diamond DAG by replacing arcs with edges. The larger the cycle is, the higher-dimensional integral will be needed to compute the arguments of conditional copulas corresponding to arcs of v-structure at a node. Note that if all copulas are Gaussian the required margin can be computed from the correlation matrix which contains parameters of this model.

If we add an arc U2 →U3 to the DAG in Figure 6 (b) (we denote the corresponding copula as $c_V^{b\prime}$ then there exists an assignment of (conditional) copulas that will not require any integration. In this case, the density is:

$$c_V^{b\prime}(u_1, u_2, u_3, u_4) = c_{12}(u_1, u_2)c_{13|2}(u_{1|2}, u_{3|2})c_{23}(u_2, u_3)c_{24}(u_2, u_4)\, c_{34|2}(u_{3|2}, u_{4|2}).$$

However, if we decided instead of c_{23} and $c_{13|2}$ to specify c_{13} and $c_{23|1}$, then $u_{3|2}$ would need to be computed via integration again. In short, because of the v-structure at node U4, the copula c_{23} is required. Hence, this copula has to be specified, for example by adding the arc U2 →U3. Then the conditional copula can then be assigned to the arc U1 →U3. Finally, let us look at Figure 6 (c). In this case, we see that the v-structure at node

U3 "interacts" with two additional v-structures: one at node U4 and one at node U5. The v-structure at U3 requires the assignment of copulas c_{12} and $c_{23|1}$ or c_{23} and $c_{13|2}$. To avoid integration in the v-structure at U4, the specification of c_{13} is required; but at the same time, integration in the v-structure at U5 can be avoided only if c_{23} is known. This creates a problem, since both c_{13} and c_{23} cannot be assigned independently at the same time.

The latest development in the theory of PCBNs (Derumigny et al., 2024) is that the structures presented in Figure6 (b) and (c) are the only problematic type of structures. If the graph G does not contain active cycles (structures as in Figure 5 (b)) and interfering v-structures, than there exists an order of parents such that the integration is not needed. The full characterizations of graph structures that are computationally efficient is proven, estimation procedure for restricted PCBN have been developed. The model will be implemented shortly as an open source package in R.

2.3. Pair-Copula BNs to asses sea level rise (SLR)

As an illustration of PCBNs with other then Gaussian copulas we present below a recent application of these models to compute sea level rise contributions from ice sheets (Bamber et al., 2019). More extensive treatment of PCBNs for this application is presented in (Kurowicka et al., 2024). The expert judgment study following protocols of (Cooke, 1991) have been performed with twenty two experts. The assessments concerned contributions to see level rise (excluding the baseline values for 2000-2010) due to Accumulation (A), Runoff (R) and Discharge (D) for Greenland (G), West (W) and East (E) Antarctic ice sheets in two temperature scenarios Low and High (rising and stabilizing in 2100 at +2° C or +5° C, respectively) in years 2050, 2100, 2200 and 2300. Experts provided their answers in the form of 5%, 50% and 95% percentiles of quantities of interest (see an example below).

*In the case of **Greenland**, for a global mean annual Surface Average Temperature rise of **3°C** by **2100** with respect to pre-industrial, what will be the **integrated contribution,** in **mm** to **SLR** relative to 2000-2010 of the following:*
i) accumulation
5% value: ___________ 50% value: ___________ 95%value:: ____________
ii) runoff
5% value: ___________ 50% value: ___________ 95%value:: ____________
iii) discharge
5% value: ___________ 50% value: ___________ 95%value:: ____________

Similar questions concerned West and East Antarctica and different temperatures up to 2200 were asked. Experts provided also answers to so called seed questions, which were used to assess their performance and choose weighted combination of experts to obtain distributions of see level rise contributions for different scenarios. Following the procedure of the Classical Method (Cooke, 1991) it has been obtained that eight experts gotten non-zero weight, where the highest weights equal to 0.28 and 0.3 were of expert 3 and 14, respectively.

All experts also quantified the dependence between these quantities at 2100 with 5° C warming with respect to pre-industrial. Experts were asked to assess bivariate dependencies between pairs presented in Figure 7 in form of the 0.5 and 0.95 exceedance probabilities ($P(Y > q_{y,\,0.5} \mid X > q_{x,0.5})$, $P(Y > q_{y,\,0.95} \mid X > q_{x,0.95})$). Some experts were comfortable enough to provide assessments in the form of values of these probabilities, others preferred to use classification presented in Table 1. All experts' answers were translated to the classification scheme in Table 1 (this classification differs slightly from the one presented in (Bamber et al., 2019)).

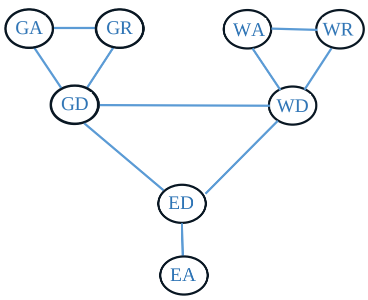

Fig. 7. Bivariate dependencies assessed by experts.

Table 1. Colour coding dependencies.

	50% ExProb	95% ExProb
Strong positive	0.8	0.8
Positive	0.7	0.5
Weak positive	0.6	0.3
Independent	0.5	0.05
Weak negative	0.4	0.03
Negative	0.3	0.02
Strong negative	0.2	0.01

Table 2. Copulas (with parameters) corresponding to colour coding.

50% ExProb	95% ExProb	Copula (param)	50% ExProb	95% ExProb	Copula
[yellow]	[yellow]	Ind	[purple]	[purple]	Gum(1.25)
[purple]	[yellow]	Frank(1.8)	[purple]	[magenta]	Gum(1.5)
[magenta]	[yellow]	Frank(2.2)	[magenta]	[magenta]	Gum(1.65)
[yellow]	[purple] [magenta]	t(0,1)	[red]	[purple]	SClay(2)
[cyan]	[yellow] [cyan]	Gauss(-0.2)	[magenta]	[red]	SClay(2.5)
[magenta]	[purple]	Gauss(0.6)	[red]	[red]	SClay(3)
[red]	[magenta]	Gauss(0.8)			

The copulas corresponding to assessed 0.5 and 0.95 exceedance probabilities are presented in Table 2. We can see that Frank's copula is chosen when no tail dependence is indicated, strong tail together with strong overall dependence corresponds to the Survival Clayton copula, and medium tail and overall dependence is associated with either Gaussian or with Gumbel copulas. In case when experts assessed 0.5 exceedance probability to be equal to 0.5 and indicated that the tail dependence is present the t copula is chosen. To quantify dependencies in PCBN the conditional copulas are computed (out of 5 types of copulas discussed above) such that the 0.5 and 0.95 exceedance probabilities for bivariate margins specified through the conditional copula are as close as possible to the ones specified by the expert. The results for two experts with the highest weights are presented in Figure 8. We can observe that both experts indicted additional independencies between variables.

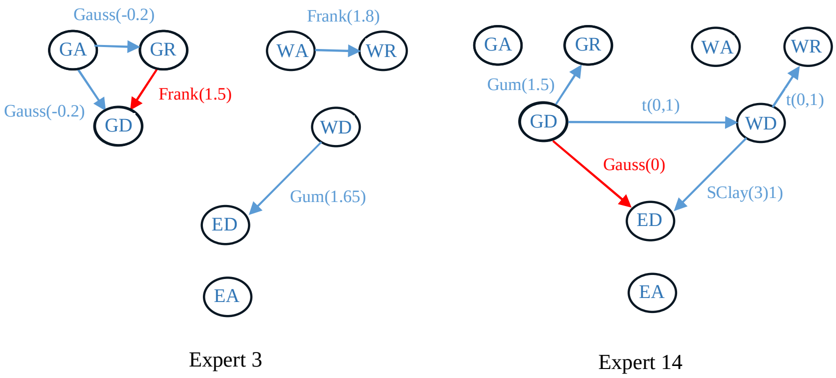

Fig. 8. Pair copula BNs with copulas (blue) and conditional copulas (red) assigned to arks for two experts with highest weights (experts 3 and 14).

The results of Icesheets contributions to see level rise by 2100 with +3^{o}C warming in [mm] for the case without and with dependence included are shown in Table 3. For comparison also the equal weight combination of experts assessments without dependencies is shown.

Table 3. The summary statistics of icesheets contribution to sea level rise for the case of +5˚C global warming by 2100in [mm]. EW (PW) denotes the equal (performance based) combination of experts' assessments (for 8 experts) and Indep (Dep) means that the independence (BN dependence) was used.

	Mean	StDev	5%	50%	95%
EW Indep	622.9	575.5	2.3	460.5	1728.1
PW Indep	586.9	468.0	12.6	482.7	1481.0
PW Dep	584.8	551.5	-4.4	434.1	1646.2

Note that due to changes in the way the copulas and their parameters are chosen as well as the different graphical representation of the dependencies (vine copulas were used in (Bamber et al., 2019)) the results of performance based combination of experts with dependence differ slightly from the one presented in (Bamber et al., 2019). We can see that there are changes in the distribution of icesheets contributions to sea level rise due to dependencies recognized by experts.

3. Conclusions

In this paper BN models, which found numerous applications in many areas, have been introduced. Their popularity stems for the intuitive graphical representation of the problem, mathematical correctness and flexible computer implementations. BNs are used in the data rich applications were one is able not only to estimate parameters of these models but also search for the graph structure supported by the data. They can also be applied in cases when not only structure but also parameters have to be assessed from experts. The flexibility of BNs to perform well in various situations as well as natural extensions to dynamic modelling environments made these models a valuable tool that engineers embraced or should embrace in the future. New developments in copula based BNs have been presented in this paper. The extensions of this kind of BNs to allow for non-gaussian types of dependencies is new and the implementation of PCBNs is not yet freely available. When ready we expect them to have a significant impact in safety and reliability applications.

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