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Deceptively Simple Method For Uncertainty Quantification And Management At AIRBUS

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Abstract

Uncertainty quantification and management (UQ&M) has become an integral part of the design for complex design processes. We describe a process developed at AIRBUS to assist during the conceptual design process. A number of work packages (WPs) are developed concurrently to satisfy evolving constraints on the uncertainty distribution of the sum of the weights of all WPs. The chief engineer has to issue uncertainty targets to the individual WPs, called margin setting, to evolve the conceptual design into compliance with tightening constraints. The focus of this article is the assessment of dependence between WP uncertainties and the impact of dependence on compliance. Although it is not possible to provide the chief engineer with an algorithmic solution for margin setting, mathematical uncertainty modeling provides important and useful insights.

Keywords: uncertainty quantification, uncertainty management, expert judgment, sensitivity analysis

1. Introduction

This article describes a process for uncertainty quantification and management (UQ&M) developed by the National Institute of Aerospace and its team members at Georgia Tech in concert with the Airbus design team. The basic design process for a new aircraft is sketched in Figure 1. From feasibility to qualification, the design passes through a number of maturity gates (MG). The entire process may consume several years and involve a commitment of a significant portion of company assets. A design flaw at an early stage can have very serious consequences if it passes undetected to late phases.

Because the initial conceptual design evolves rapidly, the UQ&M must evolve concomitantly while consuming a modest amount of resources. Because qualifying and evolving a high dimensional joint distribution can, in principle, be enormously complex; a major challenge is finding an appropriate complexity level that captures the relevant information and can be articulated with greater fidelity as the design itself becomes more detailed. The general picture of the UQ&M convergence path is shown in Figure 2.

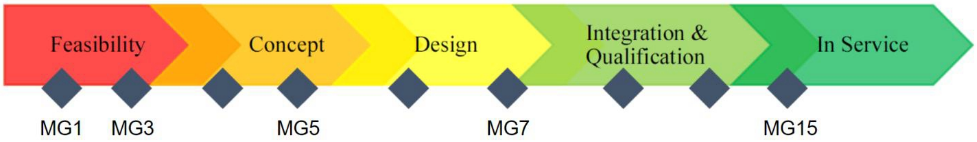

Fig. 1. Design process.

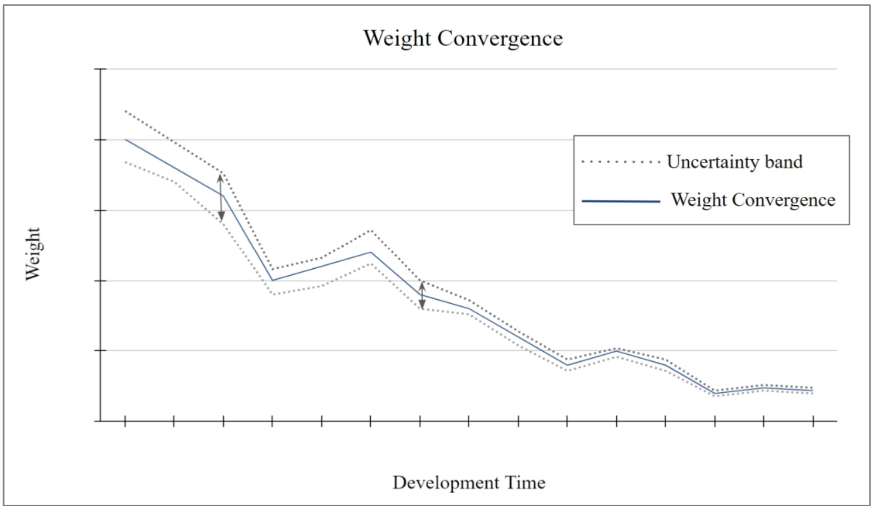

Fig. 2. UQ&M convergence.

Weight is a primary driver in aircraft design. Hence the development of weight projections will interact with the conceptual design at each stage such that the probability of hitting the final weight targets moves upward as the design evolves and the uncertainties in individual component weights shrink. Passing through the maturity gates requires that the probability of exceeding the target weight should be less than a level that is prescribed at each maturity gate.

The weight of the aircraft is the sum of the weights of its components. The number of components is large, but at early design stages, they can be grouped into 52 subsystems or work packages(WP) as shown in Table 1. The uncertainties in these WP weights have significant interactions. For example, if WP-16 (Wing Structure 3) needs to be heavier than originally estimated, it is likely that WP-19 (Wing Structure 6) will also get heavier. In this article, we describe an operational method for managing the uncertainties in the weights of WPs in Table 1 while accounting for dependencies. The following section describes weight uncertainty assessment (WUA), the system for assigning marginal distributions to these weights. Section 3 examines the impact of dependence on the weight uncertainties, and describes a system for quantification and management. The purpose of UQ is to enable the chief engineer to set margins, that is, to communicate to WP leaders how their weight distributions should change in order to pass the next maturity gate. Section 4 looks at margin setting and at performance as measured by realized values at MG13. Section 5, explores mathematical tools to aid the margin setting. A final section gathers conclusions and recommendations.

2. Weight Uncertainty Assessment (WUA)

In this section, we briefly discuss the process of estimation of marginal distributions of WPs which is detailed for the wing box model in (Reis et al., 2018). For each WP its weight is represented by a probability density function with the mean μ_i and the standard deviation $\sigma_i = ½(1 - RL_i)\mu_i$ where RL_i denotes the reliability level assumed for different estimation levels (e.g. for analytically estimated load the reliability level is 0.95, for mature load 0.97 and for supplied data 0.93). The process is divided into two stages, first, the Primary Weight is modelled with an extensive simulation study that imposes a set of static loads and flight conditions to the wing model and calculates stresses on the different parts of the wing. The Secondary Weight, which deals with non-structural weight estimation, in the case of the wing box example in (Reis et al., 2018), is modelled using Torenbeek's weight equations (Torenbeek, 1976). The results of WUA for the first twenty heaviest WPs are shown in a bar plot in Figure 3, where the black dot represents μ_i and the bar length is $2\sigma_i$. We can observe that the WP-50 and WP-45 are the largest contributors to the total weight. The remaining WPs not shown in Figure 2 have a relatively small contribution to the total weight at this stage.

Table 1. Work Packages (note there is no WP-2).

Work Packages		
WP1 Wing Controls 1	WP19 Wing Structure 6	WP36 Vertical Tailplane 1
WP3 Wing Controls 2	WP20 Wing Structure 7	WP37 Vertical Tailplane 2
WP4 Wing Controls 3	WP21 Wing Structure 8	WP38 Vertical Tailplane 3
WP5 Wing Controls 4	WP22 Wing Structure 9	WP39 Vertical Tailplane 4
WP6 Wing Controls 5	WP23 Wing Structure 10	WP40 Horizontal Tailplane 1
WP7 Wing Structure 1	WP24 Wing Structure 11	WP41 Horizontal Tailplane 2
WP8 Wing Kinematics	WP25 Wing Structure 12	WP42 Horizontal Tailplane 3
WP9 System 1	WP26 Wing Structure 13	WP43 Horizontal Tailplane 4
WP10 System 2	WP27 Wing Structure 14	WP44 Pylon
WP11 System 3	WP28 Wing Structure 15	WP45 Power Unit 1
WP12 System 4	WP29 Wing Assembly	WP46 Power Unit 2
WP13 System 5	WP30 Wing Structure 16	WP47 Power Unit 3
WP14 System 6	WP31 Fuselage Structure 1	WP48 Nose Landing Gear
WP15 Wing Structure 2	WP32 Fuselage Structure 2	WP49 Main Landing Gear
WP16 Wing Structure 3	WP33 Fuselage Structure 3	WP50 Cabin
WP17 Wing Structure 4	WP34 Fuselage Structure 4	WP51 System 7
WP18 Wing Controls 5	WP35 Fuselage Structure 5	WP52 System 8

3. Dependence

This section focuses on the dependence between the WPs with marginal distributions given in Section 2. The distributions in Section 2 are all Gaussian by default. The margin setting mentioned in the introduction can alter the WP distributions such that they are not Gaussian. However, in the analysis sketched here the "copula" will be assumed to be Gaussian. This means that the joint distribution is assumed to be derived from a Gaussian by a transformation of the marginal distributions. This assumption can also be relaxed at great computational expense, but such relaxations are not contemplated here.

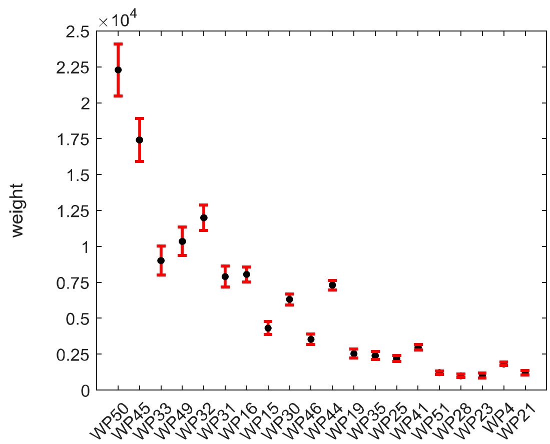

Fig. 3. Bar plot of marginal distributions of heaviest WPs.

3.1 Importance of dependence

The first task is to examine whether dependence is a potential issue. A rough cut analysis assigns various constant correlations between all WP's and compares the results for the sum of all weights, as shown in Table 2.

Table 2. Rough cut constant correlations impact on sum weight [kg].

	Independent	All 0.1	All 0.2	All 0.5	All 0.8
mean	1.4851E+5	1.4851E+5	1.4851E+5	1.4851E+5	1.4851E+5
Standard deviation	3.1897E+3	4.8775E+3	6.1160E+3	8.8461E+3	1.0913E+4
5%	1.4326E+5	1.4049E+5	1.3845E+5	1.3396E+5	1.3056E+5
95%	1.5375E+5	1.5653E+5	1.5857E+5	1.6308E+5	1.6646E+5

If the WP weights are all independent, the mean sum weight is 148,500 kg with a standard deviation 3190 and 5 and 95 percentiles as indicated in Table 2. Imposing a weak constant correlation of 0.2 has the effect of doubling the standard deviation of the sum weight. A constant correlation of 0.8 increases the standard deviation by nearly a factor 3. If the potential uncertainty increase as a result of the dependencies in Table 2 is judged acceptable, one may forego detailed analysis and conservatively assume a strong global correlation as default. In this case, the conservative default option is not acceptable. The increment of the 95th percentile from 154,000 to 166,000 kg is too large and would cast a pall over the probability of meeting the target, triggering unnecessary modifications to the conceptual design.

3.2 Dependence quantification

Quantifying dependence in a 51-dimensional joint distribution is potentially prohibitively complex. The resources spent on dependence quantification must be commensurate with the time and resources available. There are 1275 correlation values that must be assigned in order to specify a joint distribution with a Gaussian copula. Moreover, these correlation values must form a positive definite matrix: the eigenvalues of the correlation matrix must all be positive. Asking engineers to assess 1275 correlations is not feasible. A qualitative-to-quantitative process is adopted to develop a dependence structure appropriately positioned between the independent case and constant strong correlation cases in Table 2. To this end, the WPs are first ordered by importance. Importance is determined by weight and weight uncertainty. All of the WPs have important uncertainties, so a good starting point is simply to order the WPs by weight. A 51 by 51 matrix is now created with these weight-ordered WPs from which a correlation matrix will be derived. To get a correlation matrix that captures the molar dependence features, experts precede column-wise, starting with the heaviest WP. The entries in each row of a given column are assigned a dependence qualifier as strong, medium, weak, or not salient. This can be accomplished by simple colour coding. Not salient means either that the dependence is not important due to the low weight of the row WP, or that it is so small as to be negligible, or that the available information is so weak that no quantification is defensible. Strong, medium, and weak entries are assigned the initial correlations of 0.8, 0.5, and 0.2. No salient cells are left blank. A fragment of the elicited qualitative matrix is shown in Figure 4.

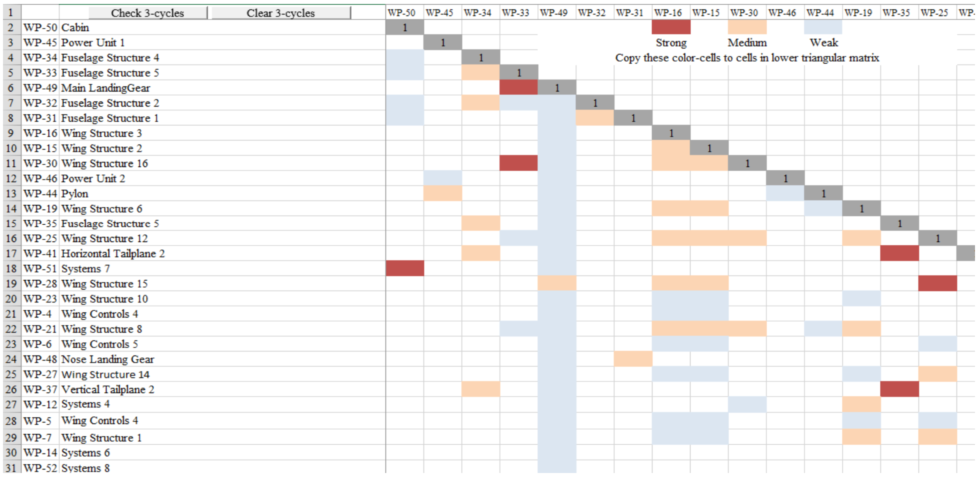

Fig. 4. Qualitative correlation matrix fragment.

The engineer responsible for populating this matrix described the experience as follows:

- Overall the experience was valuable and painless, focusing on what could be dependent elements and why. The elicitation took about two hours.
- The final result should be given a sanity check with and without dependence to see if it makes sense. Does the importance ranking of WPs make sense?
- It took some effort to synchronize the brain with the symmetrical nature of the process. The probabilistic relations are bi-directional but the causal relations are not.
- Not having to fully populate the matrix is much appreciated.
- It is good to focus on the big-weight items first.
- Using high/medium/low ratings is helpful, so as not to agonize over the details.
- It would be good to perform this with more than just one subject knowledge expert.

A typology of causes of correlations was found:

- Weight drivers are the same or dependent. An example is example loads: If loads go up on the wing then the Wing Structures WPs will all be impacted; chances are that if loads go up on the wing they will go up on the tail so HTP (horizontal tail) will be impacted.
- The weight of one WP is a driver for another. For example, landing gear mass will depend on overall aircraft mass to some degree. However, aircraft mass may be traded off against payload.
- The Weight of one WP is a secondary driver of another. A design decision may cause the bending moment carrying part of the wingbox spars to increase. Consequently, the covers' weight might be able to decrease as the spars take the load away from the covers.
- Common supplier for several WPs. Perhaps a supplier has been conservative and right at the end all that conservatism is removed or perhaps they have a dubious weight management process and suddenly surprises arrive. These may impact all the WPs they are managing.

3.2 From qualitative to quantitative

Out of 1275 correlations, the expert detected 16 strong, 39 medium, and 110 weak and left 1110 cells in the matrix unspecified. In Figure 5 the graph of WPs with strong and medium correlations represented as thick and thin edges is shown. We can see that e.g. WPs 4, 21, 22, 23, 24, which are Leading Edge subsystems are highly correlated. The partially specified 51 by 51 matrix with ones on the main diagonal has to be completed to a positive definite matrix. We used the optimization algorithm of (Qi and Sun, 2006) to find the nearest correlation matrix with the minimal square differences of off-diagonal correlations from an initial matrix. The optimization problem has been solved with a recursive, Newton-type, algorithm which requires specification of an initial matrix and the tolerance error and is implemented in the Matlab function CorrelationMatrix. A number of different settings with different initial matrices were employed to steer the optimization algorithm. They all give similar results as shown below where Matlab 0, 0.1, and 0.15 refer to results where the initial matrix in the optimization problem is such that all unspecified cells are assigned values 0, 0.1, and 0.15, respectively. The draconian outcomes in Table 2 are not supported by the more detailed engineering analysis of dependence presented in Table 3.

Table 3. Rough cut constant correlations impact on sum weight [kg].

	Independent	Matlab 0	Matlab 0.1	Matlab 0.15
mean	1.49E+05	1.49E+05	1.49E+05	1.49E+05
Standard deviation	3.19E+03	4.63E+03	5.59E+03	8.8461E+3
5%	1.43E+05	1.41E+05	1.39E+05	1.39E+05
50%	1.48E+05	1.49E+05	1.49E+05	1.49E+05
95%	1.54E+05	1.56E+05	1.5857E+5	1.58E+05

4. Sensitivity analysis and margin setting

As noted, the goal of UQ&M is to provide the chief engineer with tools for setting informed margins for the leaders of the various WPs. This involves identifying the most important contributors to the uncertainty of the sum weight. The key notion here is the correlation ratio (see e.g. (McKay, 1995)) defined in (1).

$CR(WP_i)$ =Variance over x of (Expectation of Sum Weight | $WP_i = x$) / Variance(Sum Weight) (1)

To understand this expression, suppose that the Sum Weight didn't depend on WP_i at all, as if WP_i belonged to a different airplane and was included here by mistake. The expected sum weight would not depend at all on the value of WP_i and for any value of x the expectation of Sum Weight would be the same. The variance over x of this expectation would be zero, hence also the correlation ratio. Suppose on the other hand that WP_i was completely correlated with all other WP's; then knowing the value of x would uniquely determine the value of Sum Weight. In this case the variance of the numerator would equal the variance of Sum Weight and the correlation ratio would equal 1. Intermediate cases arise when WP_i contributes to Sum Weight without completely determining it. The correlation ratio of WP_i with respect to Sum Weight will increase if the mean weight of WP_i increases and/or if the correlation of WP_i to other work packages increases. The variance of WP_i does not directly contribute to the correlation ratio, except in the case that WP_i's variance is zero, in which case WP_i is independent of all other work packages by definition. The correlation ratio, and therefore the identification of important WPs, may be strongly affected by the dependence in the joint distribution of all WPs. For example, WP_{33} (Fuselage: middle) would explain 10% of the variance in Sum Weight if all WPs were mutually independent. Taking Matlab0.1 dependence into account, WP_{33} explains 37% of this variance. Because of the correlations with other WPs, knowing the value of WP_{33} would sharply constrain the uncertainties in several other components. This does NOT mean that by changing WP_{33} we could miraculously change the weights of other components. Correlation is not causation. However, if the correlation ratio of a WP is higher than it would otherwise be if all WP's were independent, this signals that the chief engineer might pay close attention to the causes of the correlation. If the correlation is based on physical relations (e.g. other fuselage WP's, WP_{31} nose, WP_{32} fwd, WP_{34} aft) then addressing a common cause of all these weights (enhanced material, lighter wiring) would have a larger effect on the uncertainty of Sum Weight than we would surmise without taking dependence into account. On the other hand, if the correlation were due to a common supplier, then changing a supplier might have a comparable effect. However, a high value of the correlation ratio means that any change in WP_{33} might entail changes in many other WPs.

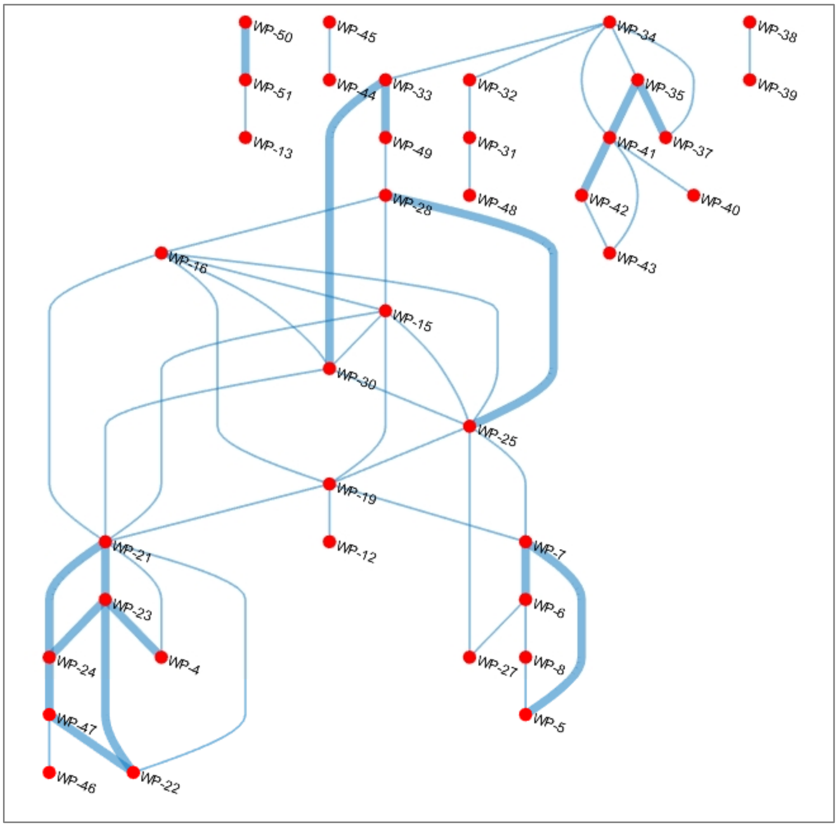

Fig. 5. Graph of WPs where edges represent strong (thick) and medium (thin) correlations assessed by the expert.

Table 4 shows the WPs ranked with respect to the correlation ratio under the Matlab 0.1 dependence model and the independent model. Notice that the importance rankings by correlation ratios are different in the independent and dependent cases. Sometimes, as in WP50, the effect of dependence is slight. In other cases the effect is substantial. Taking dependence into account, WP51 explains 24% of the variance in sum weight and is ranked 5th in importance. Without taking dependence into account, WP51 explains 0.4% of the variance and is ranked 15th.

Because of the different possible causes of correlation, and because a causal analysis cannot be performed solely on the basis of statistics, it is not possible to derive simple advice to a chief engineer on how to use correlation information in setting margins. However, a few general guidelines may be given:

1) Correlations do not by themselves affect the expected weight of the aircraft, only the uncertainty in aircraft weight.
2) In the independent case correlation ratios sum; the variance explained by WP33 and WP34 together would be the sum of their separate correlation ratios. Since the sum weight is just the sum of all the WPs, the sum of correlation ratios in the independent case of Table 4 is 1. In the dependent case, they sum to 6.1. This gives an overall indication of the importance of addressing dependence in driving down the uncertainty of the sum weight.
3) Identifying clusters of WPs with physical interdependencies and comparing their sum correlation ratio with the sum in the independent case is a good strategy for prioritization. For example, the fuselage WPs (31, 32, 33, 34) have a sum correlation ratio of 1.20 whereas their independent sum is 0.26. If we can identify and address a common cause (e.g. wiring weight), that will have an outsized effect in reducing sum weight uncertainty, relative to the independent case.
4) If the correlation causes are informational rather than physical (cause d) in Section 3), then the remediation strategies may be different, perhaps easier, than in cases with physics-based correlations.

Table 4. CR under Matlab 0.1 dependence and independence, ordered by correlation ratio.

Sensitivity: correlation with sum weight and correlation ratio					
	Dependent Matlab 0.1			Independent	
Name	Correlation	CR	Name	Correlation	CR
WP49	0.603	0.363	WP50	0.570	0.325
WP50	0.581	0.337	WP45	0.468	0.220
WP33	0.580	0.337	WP33	0.315	0.100
WP34	0.522	0.272	WP49	0.310	0.096
WP51	0.492	0.242	WP34	0.291	0.085
WP45	0.489	0.239	WP31	0.217	0.047
WP32	0.475	0.226	WP32	0.158	0.025
WP30	0.436	0.190	WP46	0.150	0.023
WP31	0.418	0.175	WP15	0.134	0.018
WP44	0.408	0.167	WP44	0.122	0.015
WP25	0.401	0.161	WP16	0.105	0.011
WP15	0.385	0.148	WP35	0.082	0.007
WP16	0.379	0.144	WP21	0.073	0.006
WP21	0.377	0.143	WP25	0.071	0.004
WP19	0.371	0.138	WP51	0.066	0.003

5. Mathematics in margin setting

As explained above, mathematics is not able to provide simple advice to a chief engineer in setting margins. Sensitivity analysis as shown in the previous section identifies which WPs contribute most to the variance of total weight. Exactly those WPs could be targeted to reduce uncertainty in total weight, e.g. reduction of the standard deviation of only WP33 by 50% would lead to the reduction of the 95 percentile of total weight by about 448 kg. A chief engineer would have to find out whether such a reduction is possible and would have to weigh its benefits and costs. In this section few additional results show how mathematics can be useful in margin setting. Mathematically the problem of a chief engineer can be structured as setting means and/or standard deviation of WPs (or groups of WPs) by changing them minimally to achieve target total weight. This formulation assumes that the dependences cannot be changed. As a measure of distance between the desired and the actual distribution the relative information (Kullback-Leibler divergence (Kullback, 1959)) can be taken. For two d-dimensional joint normal distributions f_1 and f_2 with mean vectors μ_1, μ_2 and covariance matrices Σ_1, Σ_2 , respectively, the relative information is:

$$I(f_1|\, f_2) = \frac{1}{2}\left(log\frac{|\boldsymbol{\Sigma}_2|}{|\boldsymbol{\Sigma}_1|} - d + Tr(\boldsymbol{\Sigma}_2^{-1}\boldsymbol{\Sigma}_1) + (\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1)^T \boldsymbol{\Sigma}_2^{-1} (\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1) \right),$$

where $|\boldsymbol{\Sigma}_1|, \boldsymbol{\Sigma}_1^{-1}$, denote the determinant and the inverse of $\boldsymbol{\Sigma}_1$ and T is vector transpose. Moreover, $\boldsymbol{\Sigma}_1 = D_1 R D_1$ where R is a correlation matrix and D_1 is the diagonal matrix with a vector of standard deviation, $\boldsymbol{\sigma}_1$, on the main diagonal. Hence the mathematical problem that we want to solve is:

Find vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\sigma}_1$ such that $I(f_1|\, f_2)$ is minimized, where R is Matlab 0.1 dependence, $\boldsymbol{\mu}_2$ and $\boldsymbol{\sigma}_2$ are specified via WUA, subject to target total weight (which could be the prescribed mean and variance total weight or specified 95 percentile of the distribution of total weight).

When the joint distribution of WPs is assumed to be normal constraints on variance or 95 percentile of total weight is equivalent to the 95 percentile, denoted as q_{95}, computed: $q_{95} = \mu_t + 1.96\sigma_t$ where μ_t and σ_t are mean and standard deviation of total weight. Hence all results below are shown by requiring that q_{95} or the probability of total weight exceeding 1.54E+5kg (the value when WPs were considered independent) is not larger than 0.05 is enforced for distribution of total weight when correlations are taken into account.

- Minimally informative adjustment of standard deviations of WPs keeping means fixed.
  In Figure 6 one can observe how much standard deviations of WPs would need to be decreased in case when we require that means stay unchanged and q_{95} must be decreased by 4072 kg, hence to the level of the 95 percentile of total weight when WPs are independent.

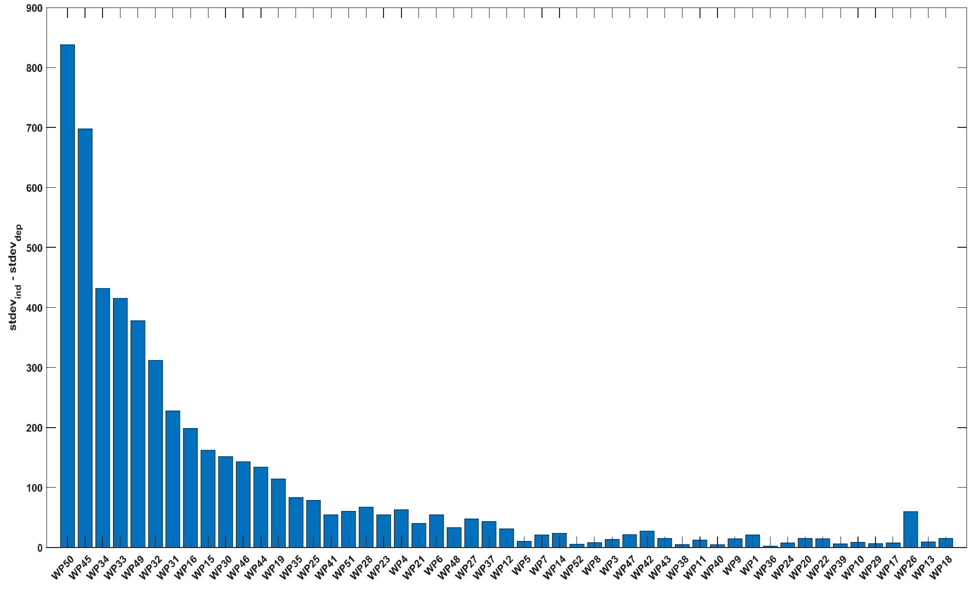

Fig. 6. Differences of means of WPs to enforce the 95 percentile of total weight equal to 1.54 E+5 (case of independent WPs).

Due to complex process of assessing uncertainty of weights of WPs in the WUA it might be unrealistic to expect that one can reduce standard deviations of WPs. Hence below we show how means would need to be adjusted to achieve the target total weight.

- Minimally informative adjustment of means of WPs keeping standard deviations fixed.

For q_{95} to be equal to 1.54E+5 instead of 1.58E+5 (when WPs are Matlab0.1 dependent) while keeping standard deviations fixed the means of WPs have to be reduced. The results are shown in Figure 7.

- Minimally informative adjustments of means of groups of WPs.

  If only the mean of WP50 (Cabin) is considered to be adjusted while keeping the standard deviation fixed, its mean would need to be reduced by 4072 kg, which is about 18% of its WUA value. However, in case one can also consider the reduction of the mean of WP51 (Airco) the minimally informative solution that would achieve the target q_{95} requires the reduction of means of WP50 and WP51 by 16.7% and 35.6% of their original value, respectively.

A chief engineer might want to consider a different, lighter design of the fuselage of the aircraft than the minimally informative solution that reduces means of WPs 31, 32, 33, 34, and 35 on average by about 10% comply with the target 95 percentile of total weight equal to 1.54E+5.

Different groups of WPs can be considered in a similar way to be adjusted to comply with the target total weight. A chief engineer would have to find out which adjustment is feasible and cost-effective. However, the analysis presented above provides a chief engineer with tools to make an informed decision to achieve desired targets at each stage of the design.

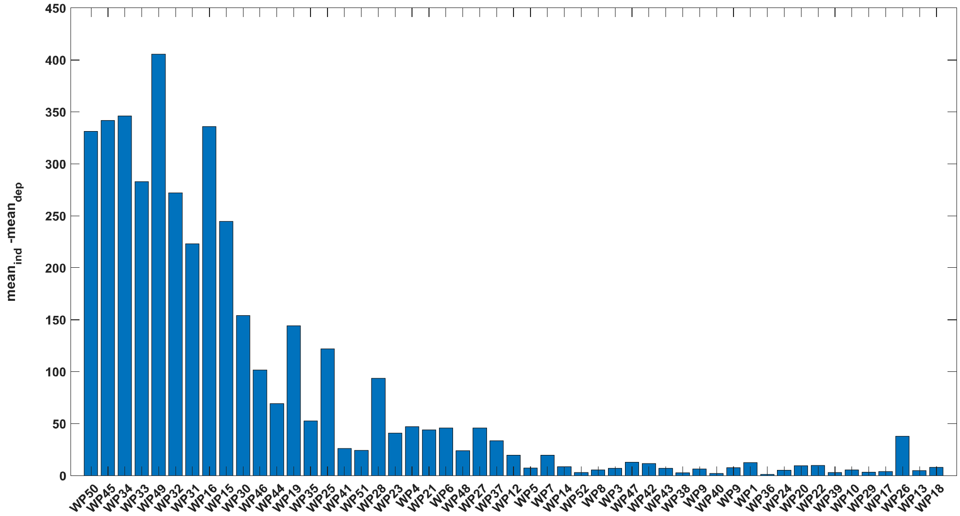

Fig. 7. Differences of means of WPs to enforce the 95 percentile of total weight equal to 1.54 E+5 (case of independent WPs).

6. Conclusions

The problem of margin setting is common to many complex design processes in which a chief engineer must communicate uncertainty reduction targets to designers of individual components in order to move the system design into compliance with system-level probabilistic constraints. In this case, the system-level constraint involved sum weight but it could just as easily apply to other measures of merit such as cost, risk, or time. This study focuses on the quantification of dependence in large problems. Although the mathematical problem is very complex, it is possible to develop qualitative assessment tools that are intuitive and sensible for the engineers and which can inform complex mathematical models. One such tool is presented here, based on the partial specification of a qualitative correlation matrix. Although the problem of margin setting cannot be reduced to a mathematical optimization problem, mathematical modeling can contribute important insights. First and foremost among these is highlighting strong dependences. Whether these dependencies are typed as a common driver, primary or secondary driver, the common supplier is critical for setting margins for the dependants. Further, recognizing dependence clusters may help in coordinating design work across WPs. Prioritizing WPs for design review should take dependence information into account. The correlation ratio, or fraction of explained variance remains an important indicator but its interpretation changes significantly once dependence is taken into account. The fractional explained variances no longer sum to unity but sum to a quantity which indicates the total dependence in the system. The importance ranking of WPs can change significantly when dependence is taken into account.

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