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Machine Learning Model To Anticipate Failures For Critical Bridge Components: Steel Rebars In Reinforced Concrete

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Abstract

With the advancement of machine learning technologies, the modeling of rebar corrosion in bridges - a critical aspect for prognosis of bridge conditions, maintenance planning and cost estimation - has become increasingly precise and reasonable. This paper introduces a novel methodology for assessing the condition of bridge rebars and estimating their Remaining Useful Life (RUL). Our approach synergizes a physics-inferred stochastic degradation model with neural networks to enhance the modeling of variabilities in the corrosion process. Initially, the rebar corrosion process is conceptualized using a two-stage Weibull-gamma degradation model, which accounts for the distinct stages of rebar corrosion based on physical scenarios. Subsequently, neural networks are employed to accurately capture and model the short-term fluctuations in corrosion levels, integrating these variations into the RUL estimation. The efficacy of our method is substantiated using lab-based experimental data, demonstrating its potential for more accurate and dynamic RUL predictions useful for maintaining bridge reliability.

Keywords: physics-infered machine learning, prediction, gamma process, remaining useful life, bridge condition

1. Introduction

Investigating the corrosion of rebars and accurately predicting their Remaining Useful Life (RUL) is of vital importance in the field of civil engineering and infrastructure maintenance. The integrity of concrete structures, notably bridges, heavily depends on the condition of the embedded rebars (Lane & Kleinhans, 2016). Corrosion in these rebars can lead to diminished structural strength, posing significant safety risks and potentially leading to catastrophic failures. Timely and precise RUL predictions enable engineers and maintenance teams to make informed decisions about repairs, replacements, and overall structural health management. This proactive approach not only ensures the safety and reliability of critical infrastructure but also contributes to optimized maintenance scheduling and cost-effectiveness (Zio, 2022). By extending the service life of structures and preventing unexpected failures, effective RUL predictions of rebar corrosion significantly enhance the resilience and sustainability of the built infrastructures.

Several studies have focused on RUL prediction. Generally, the methodologies for RUL prediction fall into two categories: physics mechanism-based models (Nicolai et al., 2007; Zhang et al., 2015) and condition monitoring-based data-driven approaches (Letot et al., 2015). Physics-based models for predicting the RUL focus on the basic principles and characteristics of degradation. One common approach is the use of stochastic processes to model the system degradation (Z. Li, et al., 2023). These models rely on random variables and parameters to represent the uncertain nature of degradation over time. In (Wen et al., 2018), a multi-phase Wiener process is established with random model parameters. Similar approaches have also been applied to model the degradation and RUL of batteries (Zhang et al., 2023) and bearings (Wang et al., 2023). Similarly, gamma process models also been applied to model monotonic degradation process such as turbine erosion (Chatenet et al., 2021), steel corrosion (Z. Li, J. Zhou, et al., 2023), and pipeline condition (Cholette et al., 2019). However, a notable challenge arises when modeling systems with multiple interrelated components. In such cases, a simple stochastic process may not suffice due to the complexity and correlations among components (Zeng et al., 2023). These correlations can stem from failure mechanisms (Fan et al., 2018; Yousefi et al., 2019), physical dependencies between components (Yousefi et al., 2020), or be represented through functional and parametric dependencies (Zheng et al., 2023). Recognizing

and accurately modeling these correlations is critical for a more comprehensive understanding and prediction of system degradation and RUL, especially in complex systems where the behavior of one component can significantly influence others. Using models that represent the system condition, the RUL can be calculated in several ways. This can be done through functions that describe the system states (Chiachío et al., 2020), deriving a random distribution (Hu & Chen, 2020), applying Karman filter (Le Son et al., 2013) or particle filter (Hachem et al., 2024), or conducting multiple simulations (Deng et al., 2020) or machine learning approaches (Yousefi et al., 2022) to predict a range of possible outcomes.

Another approach to predicting RUL does not depend on detailed models of the system's condition. This is particularly useful when the underlying mechanisms are complex or not well-understood, or when creating a system model is too complicated. In such cases, a data-driven approach is adopted. This method relies on analyzing historical and real-time data to predict RUL, bypassing the need for an explicit physical model of the system condition. These technologies include probabilistic forecasting (Aizpurua et al., 2022), support vector machine (SVM) (Tao et al., 2018), convolutional neural networks (CNN) (Y. F. Li, et al., 2023), and LSTM (Zhang et al., 2020).

While neural networks are potent tools for predicting the corrosion process in rebars, relying solely on these data-driven methods for long-term corrosion forecasting, particularly in the context of infrastructure reliability, presents several challenges. A purely neural network-based model often lacks explanatory power due to its data-driven nature and absence of underlying physical mechanisms. This can lead to issues in interpretation and understanding of the model predictions. Additionally, such models are susceptible to convergence issues as the degradation level continues to increase, potentially limiting their effectiveness for long-term predictions. To address these limitations, we developed a physics-inferred hybrid model. This approach combines the predictive power of neural networks with the explanatory and accuracy benefits of physics-based modeling, offering a more robust and reliable method for estimating the RUL of bridge rebars.

This paper is structured into five main sections to present our study. Section 2 introduces the two-stage degradation process of rebar corrosion, detailing the laboratory experiment design and data collection methods. In Section 3, we develop a novel framework that integrates this two-stage model with neural networks, enhancing the precision of modeling the corrosion process. Section 4 focuses on applying this framework to dynamically estimate the RUL of rebars, utilizing the collected data to validate the model effectiveness. Finally, Section 5 concludes the paper by summarizing our findings, discussing their implications for bridge maintenance and safety, and suggesting avenues for future research.

2. Data description

Rebar corrosion is subject to a variety of influencing factors, including environmental conditions like weather, the use of de-icing agents, the type and age of the concrete, the presence of cracks and others. To investigate the corrosion behavior of rebars, Rutgers University civil engineering laboratories conducted an extensive thirty-three-month test and study focusing on three primary factors: the type of rebar, the concentration of chloride in the environment, and the size of cracks in the concrete. Adhering to ASTM standard G109-21 (ASTM, 2005), the experiment involved setting up concrete beam specimens with induced cracks of varying sizes and depths, as illustrated in Figure 1. Each concrete specimen (280 mm in length, 150 mm in height, and 115 mm in width) embedded three steel rebars. The top rebar, exposed to a sodium chloride (NaCl) solution, experienced corrosion and functioned as the anode, while the two bottom rebars remained uncorroded, acting as Cathodes. This setup created a 'macrocell circuit', wherein the extent of corrosion on the anodic rebar intensified the ion exchange, resulting in a greater electric potential difference between the corroded and non-corroded rebars. The level of steel rebar degradation was indirectly gauged through monthly measurements of this voltage difference (in millivolts), offering a quantitative metric and insight into the typical degradation process of rebar corrosion.

The rebars used in our study are categorized into four distinct material types: Black Steel (BS), Epoxy Coated Steel (EC), Stainless Steel (SS), and MMFX. To control and vary the chloride concentration, we utilized sodium chloride (NaCl) solutions with concentrations set at 3% and 15%. As for the crack size in the concrete, we considered a range of dimensions, comprising two levels of crack width (0.011 and 0.035 inches) and two levels of crack depth (0.5 and 1 inches), in addition to a control group that featured no cracks. This diverse set of conditions was designed to comprehensively assess the impact of these variables on rebar corrosion. Figure 2 shows the collected corrosion data for MMFX rebar within first 33 months.

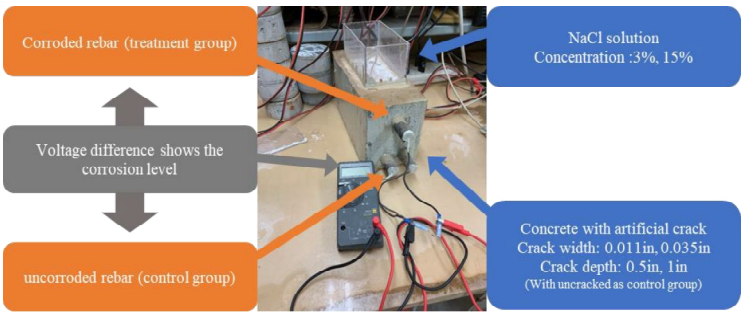

Fig. 1. Testing specimen in the rebar corrosion experiment.

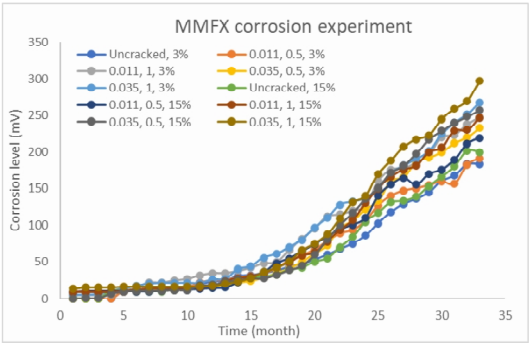

Fig. 2. MMFX corrosion paths under various conditions.

The corrosion of rebars exhibits two distinct phases, as illustrated in Figure 2, reflecting the dual-stage nature of the corrosion mechanism. In the first phase, known as the corrosion initiation stage, chloride ions gradually penetrate the concrete and any protective layers on the rebar, such as coatings. During this time, the concentration of iron ions at the steel surface remains relatively low, resulting in a slower rate of corrosion. The duration of this initial stage varies, influenced by factors like the type of concrete, the depth at which the rebar is embedded, and the nature of the rebar coating. Eventually, enough ions reach the steel surface, enabling the corrosion reaction to stabilize and progress into the second phase: the corrosion propagation stage. Here, the rate of corrosion accelerates significantly. Due to this two-stage corrosion process, traditional one-stage degradation models are inadequate. Therefore, we employ a two-stage Weibull-gamma degradation model to more accurately characterize the rebar's degradation process.

3. Neural network inferred two stage degradation model

In Section 3, we embark on an in-depth exploration of the neural network-inferred two-stage degradation model. This innovative approach integrates the robustness of physics-based models with the adaptability and precision of neural networks. Firstly, in Section 3.1, we lay the foundation with a detailed description of our physics-based two-stage model, which captures the fundamental mechanisms of rebar corrosion in its initial and propagation stages. Building on this, Section 3.2 will introduce a novel application of neural networks, not as standalone predictive tools, but as a means to refine and correct the physics-based model. This synergy aims to leverage the strengths of both approaches: the reliability and explanatory power of physics-based modeling and the dynamic adaptability of neural networks to real-world data variances. This combined methodology promises a more accurate and interpretable model for predicting the RUL of bridge rebars.

3.1. Physics-based two-stage model

Informed by the mechanism of rebar corrosion, we have implemented a two-stage Weibull-gamma degradation model to gain insights into the rebar corrosion process. The initial stage of corrosion is characterized using a time-to-event distribution, specifically the Weibull distribution, to model the duration of this initial phase. Essentially, this phase encompasses the period during which chloride ions penetrate the concrete and any protective layer on

the rebar. We represent this time span as a random variable, T_1, which adheres to the Weibull distribution., $T_1 \sim Weibull(\eta(a, C), \gamma)$. Given that this initial corrosion stage is influenced by factors such as the size of the concrete crack size, denoted as, a, and chloride concentration above the concrete, represented by C, we have configured the Weibull scale parameter η to be a function of these two variables. To be more specific, the pdf of the T_1 can be given as:

$$f(T_1 = t) = \frac{\gamma}{\eta_0 (a/a_0)^{b_1} (C/C_0)^{b_2}} \left(\frac{t}{\eta_0 (a/a_0)^{b_1} (C/C_0)^{b_2}} \right)^{\gamma - 1} e^{-\left(\frac{t}{\eta_0 (a/a_0)^{b_1} (C/C_0)^{b_2}} \right)^{\gamma}} \quad (1)$$

Where a_0 represents the benchmark level of the concrete crack size, C_0 represents the benchmark level of the chloride concentration. Thus, the expected time span of the first corrosion stage, $E(T_1)$, can be obtained as:

$$E(T_1) = \eta_0 (a/a_0)^{b_1} (C/C_0)^{b_2} \Gamma\left(1 + \frac{1}{\gamma}\right) \quad (2)$$

Upon the completion of the first stage of corrosion, the corrosion level attains a specific value, which we designate as the 'alarm threshold' and denote by x_0. This threshold marks the transition from the initial corrosion stage to the second stage, known as the corrosion propagation stage. In this subsequent stage, the corrosion process accelerates, starting from the established level of x_0 and continuing at a significantly higher rate. This progression persists until it reaches the failure threshold, denoted as H. At this critical point, the functionality of the rebars is significantly compromised, necessitating immediate replacement to ensure structural integrity. Thus, the expected corrosion level during the first stage can be given as:

$$E\big(X(t)\big) = \frac{x_0 t}{E(T_1)} = x_0 t \left[\eta_0 (a/a_0)^{b_1} (C/C_0)^{b_2} \Gamma\left(1 + \frac{1}{\gamma}\right) \right]^{-1} \quad , t \le T_1 \quad (3)$$

During the second corrosion stage, known as the corrosion propagation stage, the corrosion level intensifies, ranging between x_0 and *H*. At this juncture, a significant number of iron ions have reached the steel's surface and actively engage in the corrosion reaction. Consequently, the corrosion rate escalates, influenced significantly by the environmental chloride concentration. To accurately model this stage of degradation, employing a linear gamma process is an appropriate choice. We adopted a linear gamma process to characterize the second corrosion process, the monthly increment of corrosion level can be shows as:

$$f_X(t_{i+1} - t_i) = \frac{\beta_0 e^{b_3 (C - C_0)^{\alpha (t_{i+1} - t_i)}} x^{\alpha (t_{i+1} - t_i) - 1} exp(\beta_0 e^{b_3 (C - C_0)} x)}{\Gamma(\alpha (t_{i+1} - t_i))} \quad (4)$$

Where C_0 denoted the baseline of the chloride concentration level; i is greater than the first month of the second corrosion stage. $\alpha(t) = \alpha t$. The expected corrosion level during the second corrosion stage can be given as:

$$E\big(X(t)\big) = \frac{\alpha (t - T_1)}{\beta_0 e^{b_3 (C - C_0)}} + x_0, \quad t > T_1 \quad (5)$$

Utilizing the Weibull-gamma two-stage degradation model, we are equipped to effectively characterize the rebar corrosion process from both a physics-based mechanistic perspective and a stochastic process standpoint.

3.2. Neural network enhancement for physics-based model

While the physics-based two-stage model offers valuable insights into the rebar corrosion process, it predominantly views the phenomenon through the lens of expected corrosion values. Once environmental conditions and parameters are defined, all predictions can be derived using Equations (3) and (5). However, this model falls short in capturing the real-time fluctuations and recent changes in corrosion levels that occur in practical scenarios. Additionally, the effectiveness of physics-based models can be constrained by our current understanding of the underlying mechanisms. To address these limitations, we propose the integration of a neural network with the two-stage model. This enhancement aims to provide a more accurate and dynamic prediction of the rebar corrosion level, leveraging the neural network's ability to adapt to varying conditions and refine the model's output based on observed data.

There are generally four steps to combine the neural networks with the physics-based two stage model for better corrosion level prediction.

- Step 1: parameter estimation and expected corrosion calculation. Initially, we estimate the parameters of the two-stage model using observed data. Based on these parameters, we calculate the expected corrosion level for each stage, providing a physics-grounded baseline for our further predictions.
- Step 2: error calculation. We calculate the discrepancy between the expected and the actual observed corrosion levels. This step is crucial as it highlights the limitations of the physics-based model in real-

time scenarios, thereby identifying areas where neural network intervention is necessary.

- Step 3: neural network training. Here, we train a neural network using historical prediction error data, alongside environmental conditions, to predict the current month's error. The neural network takes as input an array comprising environmental conditions, the current month, and errors from the previous three months ($e_{i-1}, e_{i-2}, e_{i-3}$), with the aim to predict the current error (e_i). This approach enables the model to adaptively learn from past inaccuracies and refine its predictions dynamically.
- Step 4: precision enhancement. Finally, we add the error predicted by the neural network back to the expected corrosion value derived from the two-stage model. This step is a critical convergence of physics-based modelling and machine learning, resulting in a nuanced and precise prediction of the corrosion level for the current month.

By doing so, we not only retain the foundational strength of the physics-based approach but also improve it with the adaptive, data-driven capabilities of neural networks. Utilizing the comprehensive four-step process outlined above, we are now equipped to accurately predict the corrosion level of rebars under specific conditions. This methodology forms a solid foundation for assessing the reliability of rebars and anticipating their RUL.

4. Dynamic estimation for RUL

To calculate the RUL of rebars, we utilize the predicted corrosion levels derived from the methodology previously discussed. The first step involves determining the corrosion rate over a specified time window, which we denote as *CR*. This rate is calculated using Equation (6), outlined below:

$$CR_i = \frac{X_P(i+l-1)-X_P(i-1)}{l} \tag{6}$$

Where CR_i represents the corrosion rate for month i. $X_p(i)$ is the predicted corrosion level at month i. The variable l represents the time span of the observation window. It's important to note that calculating the CR over a short window, such as one month, can lead to significant fluctuations. These short-term variations may not provide a reliable basis for long-term RUL estimation, as they might not accurately reflect the overall trend of the corrosion process.

When the predicted corrosion is in the first stage of corrosion, the calculation of RUL involves summing two time frames. The first is the anticipated time until the rebar reaches the alarm threshold (denoting the end of the first stage), and the second is the expected duration of the second corrosion stage. Since the corrosion is at first stage, we assume the CR at the second stage can be fully estimated by the linear gamma process. Therefore, during the initial corrosion phase, the RUL for month i can be computed as Equation (7) presented below:

$$RUL = \frac{x_0-X_p(i)}{CR_i} + \frac{(H-x_0)\beta_0 e^{b_3(C-C_0)}}{\alpha}, \quad X_p(i) \le x_0 \tag{7}$$

Once the predicted corrosion level progresses into the second stage, it becomes necessary to calculate the CR for this stage using Equation (6). At this juncture, the RUL of the rebar can be expressed as follows:

$$RUL = \frac{H-X_p(i)}{CR_i} = \frac{\left(H-X_p(i)\right)l}{X_p(i+l-1)-X_p(i+l-1)}, \quad x_0 \le X_p(i) \le H \tag{8}$$

By employing Equation (7) for the first stage and Equation (8) for the second stage of corrosion, we are able to accurately anticipate the RUL of rebars. This approach harmonizes the physics-based two-stage model with the predictive insights of neural networks, offering a comprehensive and reliable scheme for RUL estimation.

5. Numerical results

Our methodology for estimating the RUL is exemplified through a case study focusing on EC rebar, under specific conditions: a crack width of 0.011 inches, a crack depth of 0.5 inches, and an environment with a 15% sodium chloride solution concentration. The alarm threshold, x_0 for EC has been observed to be 110 mV. To construct the two-stage model effectively, we first employed a maximum likelihood estimation approach for determining the Weibull and gamma parameters. The estimated parameters, which form the foundation of our model, are detailed in Table 1.

Table 1. Weibull-gamma two stage model parameters for EC rebar.

Stage 1 parameters				Stage 2 parameters		
γ	η_0	b_1	b_2	α	β_0	b_3
12.203	18.401	-0.030	-0.045	3.163	0.191	--

The parameter b_3 cannot be estimated based on the corrosion data, either due to its relatively small effect or the variability in the data, so it is omitted. Based on the parameters in Table 1, the expected corrosion level can be calculated and plotted as solid red line in Figure 3, the prediction mean square of error (MSE) for EC under the given condition is 174.93.

To refine the predictions of the Weibull-gamma two-stage model, we employed a simple feedforward neural network with two hidden layers. The first and second hidden layers consist of 128 and 64 neurons, respectively. After undergoing training over 700 epochs, the neural network achieved convergence to a stable state. The implementation of this neural network for bias correction notably enhanced the prediction accuracy of our model. Specifically, the MSE for the neural network-enhanced two-stage model was reduced to 51.71, which represents an improvement of 70.44%. The enhanced predictive performance of the neural-network-augmented two-stage model is also visually demonstrated as green dashed line in Figure 3:

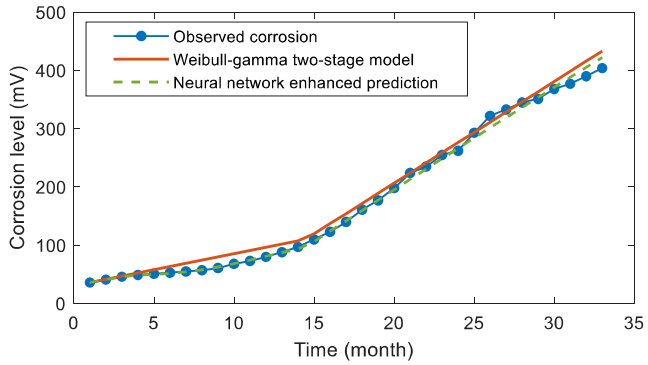

Fig. 3. Expected corrosion level prediction for EC rebar under condition [0.011, 0.5, 0.15].

To dynamically predict the RUL across varying months, we set the failure threshold, $H = 3000$ mV, and selected a window span, $l = 4$ months for calculating the *CR*. The computation of *CR* relies on the corrosion level from the preceding month, prompting us to initiate the calculation of both *CR* and RUL from the second month and continue through to the 45th month. The RUL at each of these intervals is determined using Equations (7) and (8). To illustrate these dynamic predictions, the results are graphically presented in Figures 4 and 5, offering a visual depiction of the RUL trends over the specified time frame.

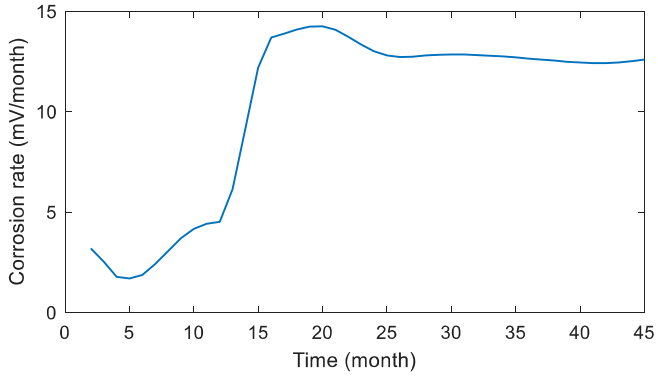

Fig. 4. Corrosion rate prediction.

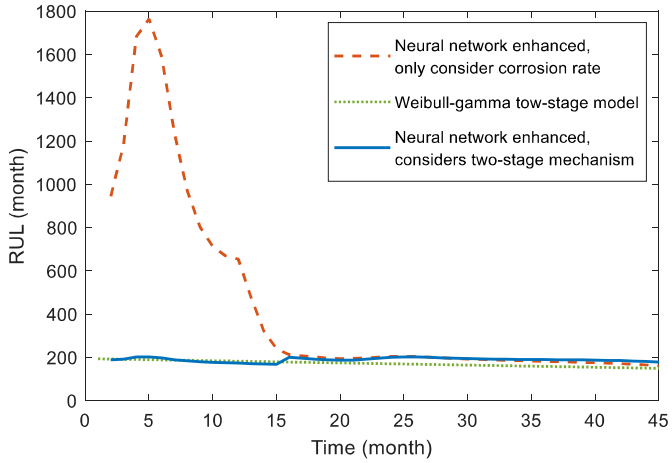

Fig. 5. Prediction for RUL considering two-stage scenario**.**

Figure 4 illustrates the predicted changes in the corrosion rate over a 45-month period. A noticeable increase in the rate is observed around the 15th month, aligning with the physical mechanism where the corrosion transitions into its second, more aggressive stage. Following this shift, the corrosion rate stabilizes, maintaining a relatively high level throughout the remainder of the period.

Figure 5 presents three distinct methods of calculating RUL. The red dashed line represents the RUL prediction based solely on the current predicted corrosion rate, which is a common approach in traditional calculations. The green dashed line illustrates the RUL estimation derived from the Weibull-gamma two-stage model prediction. In contrast, the blue solid line depicts the RUL prediction using our proposed approach, which incorporates the two-stage mechanism in both the prediction modeling and RUL estimation.

The comparison in Figure 5 reveals that when the two-stage mechanism is only considered in the modeling phase but not in RUL estimation, the result exhibits considerable variability initially, eventually stabilizing reduces over time. Conversely, relying entirely on the two-stage corrosion model for RUL estimation yields a linear prediction, failing to capture the actual variability in corrosion rates. However, by employing our proposed method, the RUL prediction achieves an optimal balance between variability and accuracy. This approach not only provides a more robust insight into the safety assessment of bridge rebars but also enhances overall decision-making in maintenance and management strategies.

6. Conclusions

This study has presented a novel approach for predicting the RUL of bridge rebars, integrating a physics-based two-stage degradation model with neural networks. Our methodology begins with a detailed examination of the rebar corrosion process, capturing its nuances through a Weibull-gamma two-stage model. This model effectively delineates the initial and propagation stages of corrosion, grounded in physical mechanisms and stochastic processes. To address the limitations inherent in purely physics-based models, particularly their inability to account for real-time fluctuations, we have introduced a neural network enhancement. This enhancement significantly improves the accuracy of the model by incorporating dynamic environmental conditions and past prediction errors into the RUL estimation process. Our results, illustrated in Figures 4 and 5, demonstrate the effectiveness of our approach. The neural network-augmented two-stage model not only aligns with the physical realities of rebar corrosion but also offers a more nuanced and adaptable prediction mechanism. By balancing the foundational strengths of physics-based modeling, with the adaptive learning capabilities of neural networks, our method provides a robust tool for assessing rebar reliability and planning maintenance strategies.

There are also several aspects for further refinement and research. One direction is the exploration of more sophisticated neural network models. Advanced architectures and learning algorithms could yield even higher prediction accuracy, adapting more effectively to complex and variable corrosion processes. Additionally, enhancing the two-stage degradation model itself presents significant potential. Incorporating random effects into the model would bring it closer to real-world conditions.

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