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# Prediction Method Of Remaining Useful Life of Rolling Bearings Based On Hybrid Degradation Model

Mengqi Huang[a], Zhengyu Du[a,b], Changhong Peng[a]

[a]*School of Nuclear Science and Technology, University of Science and Technology of China, Hefei, Anhui, P.R.China*
[b]*Science and Technology on Reactor System Design Technology Laboratory, Nuclear Power Institute of China, Chengdu, Sichuan, P.R.China*

**Abstract**

Rolling bearings are widely used, and essential components in industrial systems. Under harsh operating environments, the bearing degradation process presents complex non-linear characteristics, and the accurate remaining life prediction method plays a vital role in the stable operation and timely replacement of bearings. This paper proposes a particle filter prediction method based on a hybrid model for the complex degradation process of bearings. The particle filter method is improved based on the particle swarm optimization algorithm and the large likelihood estimation algorithm, which improves the parameter updating efficiency and prediction accuracy. A hybrid degradation model is established based on symbolic regression to discover multiple degradation models, which can eliminate the strong assumption problem caused by a single degradation model. The proposed method achieves better prediction accuracy compared to the Paris formula and can better predict the remaining life of bearings.

*Keywords*: rolling bearings, particle filter, remaining useful life.

## 1. Introduction

Rolling bearings are the core component of the rotating system, known as the "joints" of industrial machinery. It serves for a long time in a harsh operating environment. It is prone to a variety of failures, which has a significant impact on the safety and reliability of the transmission system. Accurately predicting the remaining useful life of bearings is essential for maintaining the regular operation of machinery and taking timely maintenance action to prevent serious accidents.

The remaining useful life of a bearing is the time from operation to failure, and the methods for predicting remaining useful life are usually divided into physical model-based methods, machine learning-based methods and statistical methods. Physical model-based methods include the Paris formula (Paris et al., 1963), Forman formula (Forman et al., 1967), and cumulative damage formula (Li et al., 2021) for studying crack propagation of components. For degradation problems with precise failure mechanisms, their prediction accuracy is high. Still, for complex systems, it is difficult to establish accurate physical models, and the existing physical models are difficult to generalize, significantly limiting the application of physical models.

Machine learning-based methods, such as LSTM (Ma et al., 2020), RF (Patil et al., 2018), SVR (Wang et al., 2015), etc., discover the degradation process by learning from a large amount of data and can model multi-dimensional, complex, and non-linear systems without an accurate physical model. However, they require large amounts of data and can only provide point estimates of remaining useful life that do not consider the uncertainty of the results.

Statistical methods such as Wiener (Wen et al., 2018), Gaussian (Aye et al., 2017), Gamma (Xu et al., 2012) and Markov (Chen et al., 2019) processes can estimate the degradation process using assumed degradation models and update the degradation model using real-time observations without needing an accurate physical model. However, due to the strong assumptions on the degradation process, it is often difficult to match the actual degradation situation.

Particle filter is a statistical method that does not make too many assumptions about the degradation process and is suitable for non-stationary systems with non-Gaussian noise and non-linear state equations, which uses all the historical data for joint estimation of the state and model parameters and can deal with the high uncertainty in long-term prediction. It has become one of the most popular methods for dealing with the degradation of non-Gaussian and non-linear systems. Many scholars have successfully carried out equipment's remaining useful life and reliability prediction works in recent years using particle filters and their improved techniques. Qiu et al. used an improved cuckoo search particle filter algorithm to predict the state of charge of lithium-ion batteries (Qiu et al., 2020). Zhang et al. predicted the remaining lifetime of proton exchange membrane fuel cells in relation to the performance degradation recovery phenomena based on particle filter (Zhang et al., 2016). Li et al. implemented an adaptive order-based particle filter to predict the residual wear life for aviation hydraulic pumps (Li et al., 2020).

Current particle filter is usually based on a single state equation. Due to the complexity of the degradation process of bearings, which may exhibit different degradation trends in different environments, it is difficult to track the actual degradation process with a single fixed model (Cui et al., 2022, Liao et al., 2016). It is difficult to generalize a single-state equation to the complex degradation process of bearings. To overcome the limitations caused by the single state equation of particle filter, this paper proposes a rolling bearing life prediction method based on the hybrid multiple degradation models. The main contributions of this paper are: (1) using the particle swarm optimization algorithm and the maximum likelihood estimation method to improve the particle filter, which increases the efficiency of parameter updating; (2) based on the symbolic regression to discover possible degradation models from the historical data, which removes the influence of solid assumptions of the statistical model; (3) using hybrid, multiple degradation models to discover the model that best reflects the degradation state at the current moment and make an accurate remaining useful life prediction.

The rest of this paper is organized as follows: section II describes the specific process of the rolling bearing life prediction method based on multiple degradation models, including feature extraction, feature fusion, degradation models mining, and degradation process prediction. Section III analyses the XJTU-SY bearing dataset and compares the prediction effect with the conventional particle filter algorithm based on Paris formula to verify the feasibility of the proposed method. Section IV gives the conclusions of the study.

## 2. Methodology

This section describes the particle filter algorithm and its improvement, feature extraction and fusion algorithms, symbolic regression algorithm and the overall framework of the algorithm in this paper, which are used in the process of remaining useful life prediction of rolling bearings.

### 2.1. Particle filter

Particle filter is based on Monte Carlo and sequential Bayesian inference methods. It is widely used to solve the problem of predicting the degradation process of non-Gaussian, non-linear and non-smooth systems(Djuric et al., 2003). A set of state equations $X_k = f(X_{k-1}) + Q_k$ and observation equations $Y_k = h(X_k) + R_k$ can describe the system's dynamic behavior. Where $X_k$ is the state of the system at time $k$, $f(\bullet)$ is the state transfer function, $Y_k$ is the observation of the system at time $k$, $h(\bullet)$ is the observation function, and $Q_k$ and $R_k$ are the state and observation noise of the system, respectively.

The purpose of Bayesian filter is to obtain the posterior probability density $p(x_{0:k}|y_{1:k})$ of $x_{1:k}$ based on the observation sequence $y_{1:k}$. Particle filter can be done by Monte Carlo methods by approximating the posterior probability density function based on the values $x_{0:k}^{j}$ and weights $\omega_k^j$ of the $N$ particles as in Eq. 1.

$$p(x_{0:k}|y_{1:k}) \approx \sum_{j=1}^{N} \omega_k^j \delta\left(x_{0:k} - x_{0:k}^j\right) \quad (1)$$

If the particles number is sufficient, it can be assumed that the edge probability density as in Eq. 2.

$$p(x_k|y_{1:k}) \approx \sum_{j=1}^{N_p} \omega_k^j \delta\left(x_{0:k} - x_{0:k}^j\right) \quad (2)$$

Since the posterior probability density function is unknown, the particles cannot be sampled from it but must be sampled from an artificially selected significant distribution $q(x_{0:k}|y_{1:k})$ and then adjusted with weights as in Eq. 3.

$$\omega_k^j \propto \frac{p\left(x_{0:k}^j|y_{1:k}\right)}{q\left(x_{0:k}^j|y_{1:k}\right)} \quad (3)$$

The recursive relationship of the weights can be obtained from the full probability formula as shown in Eq. 4.

$$\omega_k^j \propto \frac{p(x_{0:k}^j|y_{1:k})}{q(x_{0:k}^j|y_{1:k})} = \frac{p(y_k|x_k^j)p(x_k^j|x_{k-1}^j)p(x_{0:k-1}^j|y_{1:k-1})}{p(y_k|y_{1:k-1})\,q(x_k^j|x_{0:k-1}^j,y_k)\,q(x_{0:k-1}^j|y_{1:k-1})} \propto \omega_{k-1}^j \frac{p(y_k|x_k^j)p(x_k^j|x_{k-1}^j)}{q(x_k^j|x_{k-1}^j,y_k)} \tag{4}$$

Where $q(x_k^j|x_{k-1}^j, y_k)$ is the significant density function, $p(y_k|x_k^j)$ is obtained from the observation equation, and $p(x_k^j|x_{k-1}^j)$ is obtained from the equation of state.

This is the general flow of the particle filter algorithm:

(1) Enter the last moment state value $\{x_{k-1}^j\}$, the weight value $\{\omega_{k-1}^j\}$ and the current moment observation value $y_k$, where $j = 1,2,3,\cdots,N_p$.

(2) Sample $N_p$ particles $x_k^j \sim q(x_k|x_{k-1}^j, y_k)$ from the significant density function $q(x_k|x_{k-1}^j, y_k)$ , where $j = 1,2,3,\cdots,N_p$.

(3) Assign weights $\omega_k^j \propto \omega_{k-1}^j \frac{p(y_k|x_k^j)p(x_k^j|x_{k-1}^j)}{q(x_k^j|x_{k-1}^j,y_k)}$ to each particle and normalise them.

(4) To determine if resampling is necessary, calculate the adequate sample number $\widehat{N}_{eff}$ and compare it to the threshold $N_T$. Resampling is required if the adequate number of samples is less than the threshold. The final output includes the weights $\tilde{\omega}_k^j = \frac{\omega_k^j}{\sum_{j=1}^{N_p}\omega_k^j}$ at times $k$ and the state estimate $\hat{x}_k = \sum_{j=1}^{N_p}\tilde{\omega}_k^j\, x_k^j$.

### 2.2. Improved particle filter algorithm

As the particle filter algorithm is less efficient in updating the parameters with joint estimation, it is challenging to drive the parameters to obtain the best value. Therefore, this paper proposes a particle filter parameter updating method based on the particle swarm optimization algorithm and the maximum likelihood estimation algorithm to improve the particle filter algorithm. The likelihood function shown in Eq. 5 denotes the probability that the parameter takes $\theta$ when the observation value takes $(y_0, y_1, \cdots, y_k)$.

$$p(\theta\,; y_0, y_1, \cdots, y_n) = p_\theta(y_0)p_\theta(y_1|y_0)\cdots p_\theta(y_k|y_{k-1}) \tag{5}$$

Eq. 6 to Eq. 10 derive an approximate expression of the likelihood function.

$$p_\theta(y_k|y_{k-1}) = \int_{x_k} p(y_k|x_k)p(x_k|y_{1:k-1})dx_k \tag{6}$$

$$p(y_k|x_k) = f_{R_k}[y_k - h(x_k)] \tag{7}$$

$$p(x_k|y_{1:k-1}) \approx \sum_i \omega_k^i\, \delta(x - x_k^i) \tag{8}$$

$$p_\theta(y_k|y_{k-1}) \approx \sum_i \omega_k^i \cdot f_{R_k}[y_k - h(x_k^i)] \tag{9}$$

$$L(\theta) = p(\theta\,; y_0, y_1, \cdots, y_n) \approx \prod_k \sum_i \omega_k^i \cdot f_{R_k}[y_k - h(x_k^i)] \tag{10}$$

The particle swarm optimisation algorithm simulates the foraging behaviour of bird flocks with few parameters and fast calculation, which is suitable for parameter optimisation of particle filter (Wang et al., 2018). The maximum likelihood estimation algorithm uses Eq. 10 as the likelihood function to find the parameter that makes the degraded state take the maximum probability of the true value in the whole state space(Yin et al., 2021). By combining the particle swarm optimization algorithm and the maximum likelihood estimation algorithm, the optimal parameter value $\theta_m = \underset{\theta}{argmax} L(\theta)$ that maximises the likelihood function $L(\theta)$ can be obtained in each round of calculations.

### 2.3. Feature extraction

To monitor the degradation state of bearings, it is essential to extract features from the monitoring signals, typically vibration signals. These features should reflect the degradation information of the bearings. This section covers feature extraction methods based on time-domain, frequency-domain, and time-frequency-domain. The features used to assess degradation can be categorized into three types: time domain, frequency domain, and time-frequency domain. Time domain features are calculated directly from the vibration signals (Li et al., 2014), while frequency domain features are extracted by Fourier transforming the original vibration signals (Wu et al., 2021). Time-frequency domain features are represented by the wavelet packet node energies obtained by the wavelet packet transforming the original data (Gao, 2022). Table 1 shows the degradation features used in this section.

Table 1. Bearing degradation characteristics.

| Type | Feature |
|---|---|
| Time-domain | Max |
| | Root Mean Square |
| | Absolute Mean |
| | Peak-to-peak Value |
| | Margin Factor |
| | Skewness |
| | Peak Index |
| | Pulse Index |
| Frequency-domain | Frequency Mean |
| | Frequency Concentration |
| | Frequency Root Mean Square |
| Time-frequency-domain | 1st Node Energy |
| | 2nd Node Energy |
| | 3rd Node Energy |
| | 4th Node Energy |
| | 5th Node Energy |
| | 6th Node Energy |
| | 7th Node Energy |
| | 8th Node Energy |

This section evaluated features using a composite metric that considers relevance, monotonicity, and robustness to eliminate irrelevant features with low relevance to the degradation process (Zhang et al., 2016). We classified the features into trend and random parts using a smoothing method as shown in Eq. 11.

$$x(t) = x_R(t) + x_T(t) \tag{11}$$

Where $x_T(t)$ is the trend component and $x_R(t)$ is the stochastic component.

The correlation as shown in Eq. 12 denotes the degree of correlation between the degradation characteristics and the monitoring time. The larger the value, the more noticeable the change in the degradation characteristics of the bearing over time.

$$Cor = \frac{\left|\sum_{t=1}^{L}(x(t)-\bar{x})(t-\bar{t})\right|}{\sqrt{\sum_{t=1}^{L}(x(t)-\bar{x})^2 \sum_{t=1}^{L}(t-\bar{t})^2}} \tag{12}$$

Where $x(t)$ is the degradation characteristics at time $t$, $\bar{x}$ is the mean value of degradation characteristics, $\bar{t}$ is the mean value of the time series, and $L$ is the total number of monitoring times.

Monotonicity as shown in Eq. 13 refers to the trend of increasing or decreasing degradation characteristics. The larger the value of monotonicity, the greater the continuous monotonic degradation of bearing performance.

$$Mon = \frac{\left|\sum_{t=1}^{L} diff(x) - \sum_{t=1}^{L} -diff(x)\right|}{L-1} \tag{13}$$

Where $diff$ is the difference between two consecutive moments of the feature.

Robustness shown in Eq. 14 refers to the ability of a feature to remain stable even when affected by noise interference. The greater the robustness, the more effectively the feature can demonstrate the degradation process of the bearing under the influence of noise.

$$Rob = \frac{1}{L}\sum_{t=1}^{L} exp\left(-\left|\frac{x_T(t)}{x(t)}\right|\right) \tag{14}$$

A comprehensive evaluation index $Index = 0.2Cor + 0.5Mon + 0.3Rob$that combines the effects of correlation, monotonicity, and robustness was established (Wang et al., 2023). Measure each degradation feature listed in Table 1 and select the features with the highest composite evaluation index for fusion.

### 2.4. Feature Fusion

Kernel Principal Component Analysis (KPCA) is a method for reducing dimensionality that uses a nonlinear mapping $\Phi: R^N \rightarrow F$ to map the original input vector $x_t$ into a high-dimensional feature space $\phi(x_t)$, and then computes the linear principal components in the high-dimensional feature space (Anowar et al., 2021).

KPCA requires the computation of the eigenvalue problem in Eq. 15 (Cao et al., 2003):

$$\lambda_i u_i = \tilde{C} u_i,\ i = 1, \cdots, l \tag{15}$$

Where $\tilde{C} = \frac{1}{l}\sum_{t=1}^{l} \phi(x_t)\phi(x_t)^T$ is the covariance matrix of sample $\phi(x_t)$, $\lambda_i$ is a non-zero eigenvalue, and $u_i$ is the corresponding eigenvector.

The eigenvalue problem of Eq. 15 can be converted to the problem of Eq. 16:

$$\tilde{\lambda}_i \alpha_i = K \alpha_i,\ i = 1, \cdots, l \tag{16}$$

Where $K$ is the kernel matrix, and the dot product operation is replaced by the kernel function in the high-dimensional feature space $K(x_i, x_j) = \phi(x_i) \cdot \phi(x_j)$, which can be used to deal with $\phi(x_t)$ of any dimension without explicitly calculating $\phi(x_t)$. The corresponding eigenvectors of the kernel matrix can be calculated and normalised according to Eq. 17.

After calculating and normalising the eigenvector $\alpha_i$ corresponding to the kernel matrix $K$, the principal components of $x_t$ can be calculated according to Eq. 17:

$$s_t(i) = u_i^T \phi(x_t) = \sum_{j=1}^{l} \tilde{\alpha}_i (j) K(x_j, x_t),\ i = 1, \cdots, l \tag{17}$$

KPCA has numerous applications in feature reduction and fusion. This section employs kernel principal component analysis to downscale the filtered degraded features and fuse them into health factors reflecting the bearings' degraded nature.

### 2.5. Symbolic Regression

Symbolic regression is a supervised machine learning technique to discover a hidden mathematical expression or function that best fits the relationship between inputs and outputs in a given dataset(La et al., 2021). Unlike traditional regression methods, symbolic regression constructs a mathematical expression by searching and combining basic mathematical operators and functions rather than just finding the parameters of a mathematical model. By performing symbolic regression operations on the bearing dataset, it is possible to identify potential degradation models and to model degradation appropriately for bearings in specific operating environments.

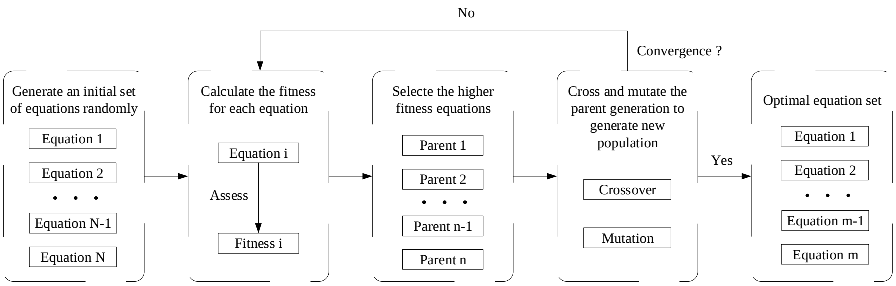


Fig. 1. Flow of symbolic regression to uncover potential degradation models.

### 2.6. Hybrid Models to Generate RUL Predictions

There are multiple degradation models generated by symbolic regression at each moment in time, and traditional particle filter algorithms use only one equation of state to model the entire degradation process. Due to the high nonlinearity and complexity of the bearing degradation process, it is difficult to accurately model the entire degradation process using a single degradation model. Therefore, competitive modeling using multiple degradation models can solve this problem caused by a single model. When predicting the RUL at moment $t_k$, assume that there are $n$ alternative degradation models and use $n$ degradation models to predict the future state of the bearing respectively. After obtaining the observed values at time $t_{k+1}$, the prediction results of degradation model $n$ are evaluated by Eq. 18.

$$Error_i = \left| y_{k+1}^i - y_{k+1} \right|, i = 1, \cdots, m \quad (18)$$

The result of the degradation model with the lowest error is taken as the RUL of the bearing at the moment $t_k$.

$$RUL = L - t_k, y_{t+L} < failure\ threshold \le y_{t+L+1} \quad (19)$$

The method proposed in this paper is illustrated in the figure below. Firstly, the bearing vibration dataset is analyzed in the time-frequency domain to extract multiple degradation features. Next, the degradation features are evaluated and screened based on comprehensive indexes considering correlation, monotonicity, and robustness. Using the kernel principal component analysis method, the selected features are combined to create a health factor representing the bearing's degradation state. Symbolic regression is used to identify potential degradation models from the dataset. An improved particle filter algorithm is then employed to predict the remaining service life of the target dataset, taking into account multiple degradation models.

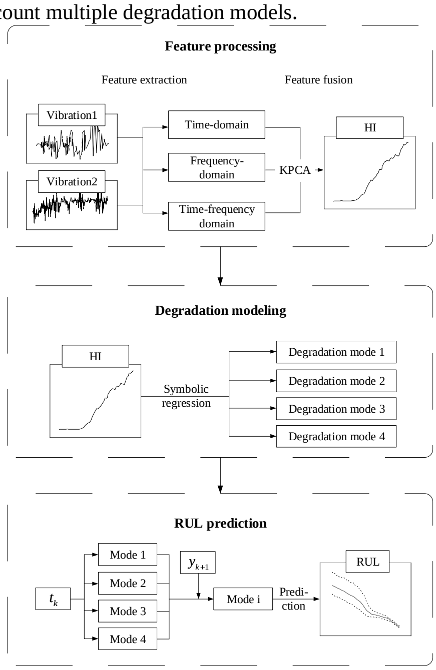


Fig. 2. Flow of bearing remaining service life prediction method based on hybrid degradation model.

## 3. Case study

This section validates the proposed method using the XJTU-SY bearing dataset (Wang et al., 2018). The platform comprises alternating current motors, motor speed controllers, rotating shafts, support bearings, hydraulic loading systems, and test bearings. The test includes three types of working conditions, as shown in Table 2, with five bearings in each type of working condition. This section focuses on the first three bearings in Case 1. Bearing 1_1 and Bearing 1_2 vibration signals are used to identify alternative degradation models. The remaining service life prediction is based on the data of Bearing 1_3.

Table 2. Bearing data sets analysed in this paper.

| Operating condition | Dataset | Sample size | Actual lifetime | Failure position |
|---|---|---|---|---|
| | Bearing 1_1 | 123 | 2h3min | Outer ring |
| 1 | Bearing 1_2 | 161 | 2h41min | Outer ring |
| | Bearing 1_3 | 158 | 2h38min | Outer ring |

Fig. 3 displays the vibration data of Bearing 1_1. The data exhibits significant noise, making it challenging to directly determine the bearing degradation state. However, the data also contains information about the bearing degradation, necessitating the extraction of features that reflect the bearing degradation state.

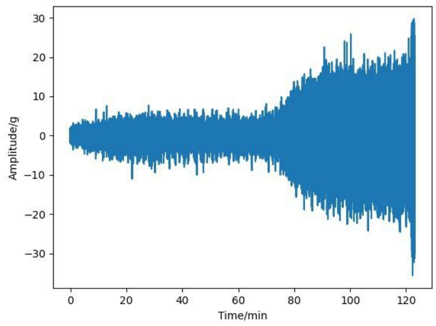


Fig. 3. Horizontal vibration data for Bearing 1_1.

The vibration data were first feature-extracted. The degraded features were evaluated based on correlation, monotonicity, and robustness. The results are shown in Table 3.

Table 3. Evaluation of features for dataset Bearing 1_1.

| Feature | | Correlation | Monotonicity | Robustness | Composite index |
|---|---|---|---|---|---|
| Time-domain | Max | 0.8926 | 0.2459 | 0.3599 | 0.4094 |
| | Root Mean Square | 0.8638 | 0.5737 | 0.3671 | 0.5698 |
| | Absolute Mean | 0.8596 | 0.5901 | 0.3673 | 0.5772 |
| | Peak-to-peak Value | 0.8905 | 0.2459 | 0.3616 | 0.4095 |
| | Margin Factor | 0.1315 | 0.0327 | 0.3618 | 0.1512 |
| | Skewness | 0.4917 | 0.2131 | 0.3648 | 0.3143 |
| | Peak Index | 0.5219 | 0.0491 | 0.3490 | 0.2337 |
| | Pulse Index | 0.3853 | 0.0327 | 0.3544 | 0.1998 |
| Frequency-domain | Frequency Mean | 0.8942 | 0.6065 | 0.3673 | 0.5923 |
| | Frequency Concentration | 0.7166 | 0.5409 | 0.3697 | 0.5247 |
| | Frequency Root Mean Square | 0.8531 | 0.5573 | 0.3668 | 0.5593 |
| Time-frequency-domain | 1st Node Energy | 0.6993 | 0.2561 | 0.3411 | 0.3703 |
| | 2nd Node Energy | 0.6268 | 0.2066 | 0.3680 | 0.3390 |
| | 3rd Node Energy | 0.7109 | 0.4876 | 0.3662 | 0.4958 |
| | 4th Node Energy | 0.8609 | 0.3719 | 0.3669 | 0.4682 |
| | 5th Node Energy | 0.9016 | 0.4545 | 0.3666 | 0.5175 |
| | 6th Node Energy | 0.8949 | 0.5702 | 0.3668 | 0.5741 |
| | 7th Node Energy | 0.8968 | 0.5041 | 0.3660 | 0.5412 |
| | 8th Node Energy | 0.8892 | 0.4545 | 0.3662 | 0.5150 |
| KPCA | | 0.9004 | 0.7213 | 0.3743 | 0.6531 |

The degradation features are screened by taking the comprehensive evaluation index higher than 0.5. The screened features are fused using kernel principal component analysis, and the results show that the fused features have better properties than the single degradation features. Based on the K-means clustering algorithm, the whole degradation process of the bearing is divided into the normal operation stage and the degradation stage(Na et al., 2010). The remaining life prediction is not carried out in the normal operation stage, and the remaining life prediction is started when the bearing operates to the degradation stage.

Learning based on symbolic regression algorithms for the Bearing 1_1 and Bearing 1_2 datasets yields a collection of alternative degradation models, as shown in Table 4. These degradation models are partially capable of modelling the degradation process of the bearings and can therefore be used as alternative degradation models for the bearing under study.

Table 4. Degenerate models obtained from symbolic regression.

| Dataset | Degradation model |
|---|---|
| Bearing 1_1 | $y = x + x^{a}$ |
| Bearing 1_1 | $y = (x + a)^{2}$ |
| Bearing 1_2 | $y = x * a$ |
| Bearing 1_2 | $y = x * a^{x}$ |

Alternative degradation models are applied to the Bearing 1_3 data set, and the predicted future state value corresponding to each degradation model is calculated at each moment and compared with the observed data at the next moment, and the degradation model with the smallest error is selected for the remaining service life prediction at that moment.

Eq. 20 shows the state equation for the commonly used Paris formula.

$$x_k = x_{k-1} + a x_{k-1}^{b} \Delta t_k + Q_k \tag{20}$$

The comparison between the results of the method proposed in this section and those predicted by the commonly used Paris formula is shown in Fig. 4 and Fig. 5. The hybrid degradation model can effectively reduce the prediction error of a single degradation model for the complex degradation process of bearings.

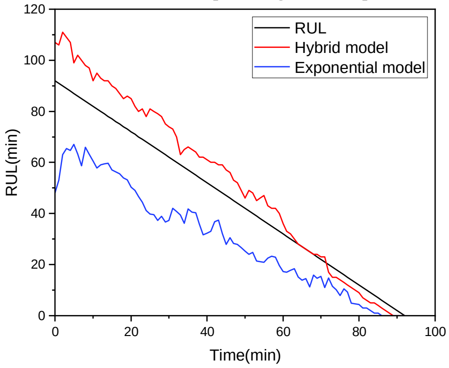


Fig. 4. Comparison of RUL predicted by hybrid model and single exponential model.

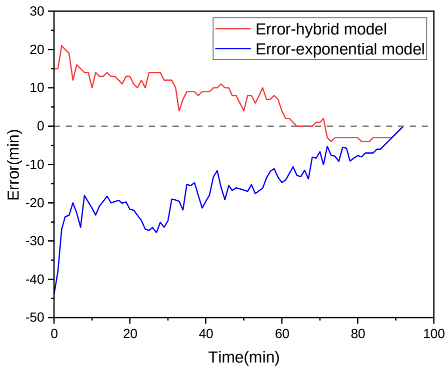


Fig. 5. Comparison of errors in predicting RUL between hybrid model and single exponential model.

Fig. 6 displays the uncertainty distribution of the remaining useful life (RUL) predicted by the method described in this section for nine time nodes (from 10 minutes to 90 minutes in 10-minute intervals). The uncertainty of the RUL prediction decreases over time due to the particle filter obtaining more monitoring data and a better understanding of the degradation process of the bearings.

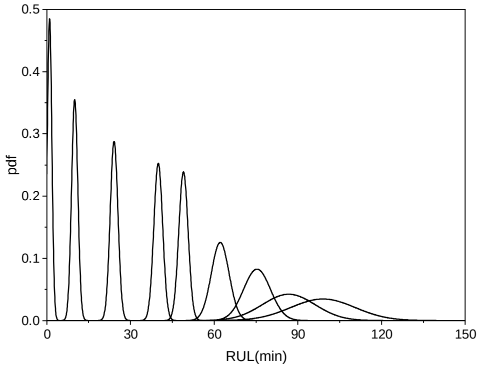

Fig. 6. Distribution of RUL uncertainty predicted by the hybrid model.

## 4. Conclusion

This paper proposes a method for predicting hybrid multiple degradation models based on symbolic regression. The aim is to address the challenge of accurately predicting a single model for bearings' complex, nonlinear degradation process. The method uses symbolic regression algorithms to learn potential multiple degradation models from the bearing dataset. This approach can improve the adaptivity of the remaining life prediction of bearings. Using multiple degradation models and observed data for judgment at the next moment can improve the bearing life prediction accuracy and eliminate the strong assumption problem associated with a single degradation model. A comparison between this method and the single-exponential model based on the Paris formula demonstrates that this method has superior prediction accuracy and ability for remaining bearing life.

## Glossary

| Glossary | Full Name |
|---|---|
| KPCA | Kernel Principal Component Analysis |
| LSTM | Long Short-Term Memory |
| PCA | Principal Component Analysis |
| RF | Random Forest |
| RUL | Remaining Useful Life |
| SVR | Support Vector Regression |
| XJTU-SY | Xi’an Jiaotong University and the Changxing Sumyoung Technology Co., Ltd. |

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