Advances in Reliability, Safety and Security, Part 3 – Kołowrocki, Dąbrowska (eds.)
© 2024 Polish Safety and Reliability Association, Gdynia, ISBN 978-83-68136-15-9 (printed), ISBN 978-83-68136-02-9 (electronic)

Advances in Reliability, Safety and Security
ESREL 2024
Monograph Book Series

Alternative Approach To Modelling Complex Failure Behaviors Of Technical Products: Comparative Study Sb Johnson Versus Weibull

Jannis Pietruschka, Alicia Puls, Stefan Bracke

University of Wuppertal, Wuppertal, Germany

Abstract

The Weibull distribution is frequently used due to its simplicity of interpretation and high flexibility in modeling technically complex failure behaviors. However, fitting a mixture of different failure behaviors poses challenges. In such cases, two distinct approaches, namely separation and mixed population methods, are commonly considered. Nevertheless, the use of these approaches increases the number of parameters, leading to decreased interpretability. As an alternative, the Sb Johnson distribution, with an additional fourth parameter, provides the capability to represent more complex distributions. However, the parameters of the Sb Johnson distribution are not interpretable in the manner of the three-parameter Weibull distribution. This paper presents a case study where the parameters of the Sb Johnson distribution are interpreted similarly to the Weibull distribution. To achieve this, three synthetic datasets with varying failure behaviors are examined, and a correlation analysis between the Sb Johnson and Weibull parameters is conducted. This study aims to provide insights into the interpretability and applicability of the Sb Johnson distribution as an alternative to the Weibull distribution in modeling complex failure behaviors.

Keywords: Sb Johnson, Weibull, Weibull mixed-population, Failure behavior, Bathtub curve

1. Introduction

The Weibull distribution model is often used to map and analyze the failure behavior of technically complex products. The distribution model according to Ernst Waloddi Weibull (1951) is characterized by advantageous flexibility and is also mathematically easy to handle. In principle, it is an exponential distribution. If the damage data from several failed products is available, a Weibull distribution model is adapted, so the failure behavior can be mapped and, for example, failure probabilities can be determined for certain values in relation to the mileage-related variable.

There are various forms of the Weibull distribution model (so-called "Weibull distribution family"); see Sec. 3.2. One advantage of this distribution model is that parameters of the respective Weibull distribution models can be easily interpreted to characterize the failure behavior of the product after adjustment to the specific damage data available. The three-parameter Weibull distribution model with the parameters failure-free time t_0 (threshold: theoretical, first failure time), characteristic service life T (location parameter: characteristic life span) and shape parameter b (gradient), which characterizes the failure behavior, is at the center of the present work. As mentioned before, with a Weibull distribution model, failure behavior, usually caused by a damage mechanism, can be mapped very well. Fitting a mix of different failure behaviors is problematic. In this case, there are two different approaches as state of the art:

- Separation of the data for the different damage mechanisms and the subsequent adaptation of different Weibull distribution models (algorithm for separating damage data cf. (Bracke and Haller, 2009),
- Adaptation of a mixed-population approach: use of an alternative or competing model (Bracke, 2024).

The advantage of a mixed-population approach is that a model can be fitted for the entire product service life including all different damage mechanisms of the corresponding phases of failure behaviors. The disadvantage is that, in addition to the increased effort for the parameter fitting, the explicit parameter interpretation is difficult, as the number of parameters increases significantly. If, for example, the bathtub curve (failure rate (3); see Figure 1) were to be modelled using a mixed population approach, three Weibull distribution models would be

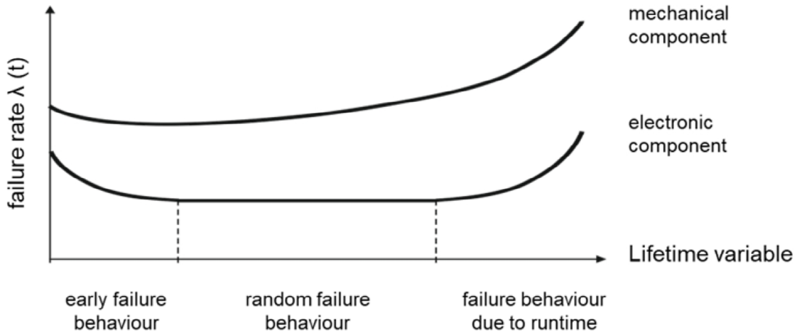

Fig. 1. Schematic visualization for mapping the failure behavior with failure rate $\lambda(t)$ of a product for different phases related to the use phase.

required for the three phases (with respect to early failure behavior, random failure behavior, runtime-related failure behavior; see (7)). This would result in at least nine or ten parameters (depending on model) that would be difficult to interpret.

An alternative is the Sb Johnson distribution model, which also is part of a distribution family. The focus in this paper is on the Sb Johnson distribution model, which is defined by (12) and comprises a total of four parameters: two shape parameters and two location parameters.

Compared to the mixed population approach, fewer parameters would therefore have to be interpreted, but at the same time there is also the potential, that the failure behavior of a product can be mapped over several phases (i.e. failure modes). The result would be a model for mapping complex failure behaviors with a manageable set of parameters compared to the state-of-the-art mixed population approach. Furthermore, there would be no need to separate the data set. The difficulty lies in the interpretation of the parameter values of a Sb Johnson model adapted to a specific damage data set in relation to the known parameter interpretations in a Weibull distribution model.

The Sb Johnson distribution model is currently only used in a few specialist disciplines to solve detailed problems; these include, for example, mapping the size distribution of raindrops (Cugerone and De Michele, 2014) or exposure assessment for epidemiological studies (Flynn, 2006). The application and interpretation of the Sb Johnson model exists only in initial research works; cf. for example (Kudus et al, 1999) and (Slifker and Shapiro, 1980).

The aim of the present research study is the application and interpretation of the Sb Johnson distribution model for mapping complex failure behaviors of technical products.

2. Goal of research works

The overall objective of this comparative study is to represent complex failure behaviors based on concrete damage data using an Sb Johnson distribution model. The following sub-goals are pursued:

1) Adaptation of Sb Johnson distribution models to represent the elementary failure behaviors: early failure behavior, random failure behavior and failure behavior due to runtime,

2) Comparative analysis with Weibull distribution models: Section wise/separated view versus mixed distribution approach,

3) Correlation analyses between parameter sets of Sb Johnson distribution models and Weibull distribution models based on representative damage data sets,

4) Interpretation of the parameterizations regarding the actual failure behavior,

5) Discussion of the advantages and disadvantages of applying an Sb Johnson distribution model versus a Weibull distribution model.

3. State of the art

3.1. Failure root causes / Failure modes

In principle, a distinction can be made between simple failure behaviors - usually based on one failure root cause - and complex failure behaviors - usually based on several failure root causes. Furthermore, a distinction is made between the elementary failure phases of early failure behavior, random failure behavior and failure behavior due to runtime (see Figure 1). Early failure behavior includes e.g. assembly errors and setting errors. Furthermore, sporadic control unit failures, for example, are assigned to the random failure behavior phase. Typical failure behaviors due to runtime are, for example, wear mechanisms, oil leaks and noise emissions. The causes of failure that lead to runtime-related failure behavior are the focus of this paper, see Section 5.1.

3.2. Modeling of failure behavior

In reliability engineering, failure behavior is modelled using distribution models. Especially the probability density function $f(x)$ (cf. (1)), the cumulative density function or failure probability $F(x)$ (cf. (2)) and the failure rate $\lambda(x)$ (cf. (3)) are used to model the failure behavior of a runtime variable x (Bracke, 2024).

$$f(x) = \frac{d}{dx} F(x) \tag{1}$$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u) du \tag{2}$$

$$\lambda(x) = \frac{f(x)}{1-F(x)} \tag{3}$$

The failure probability can be estimated by the cumulative frequency using median ranking, cf. (4). The failure probability $F(x)$ of the value x_i is calculated using a ration of the index value i to the sample size n (Bracke, 2024).

$$F(x) = \frac{i-0.3}{n+0.4} \tag{4}$$

A distribution model with wide use in reliability engineering is the Weibull distribution model (Weibull, 1951). Two forms are distinguished: a two parameter (cf. (5)) and a three-parameter form (cf. (6)). The parameters, besides the life span variable x, are shape parameter (gradient) b, location parameter T (characteristic life span) and in case of the three-parameter Weibull distribution the failure-free time (threshold) t_0.

$$F(x) = 1 - e^{-\left(\frac{x}{T}\right)^{b}} \tag{5}$$

$$F(x) = 1 - e^{-\left(\frac{x-t_0}{T-t_0}\right)^{b}} \tag{6}$$

The advantage of the Weibull distribution is its flexibility regarding the modelled failure behavior. By variating parameter b, different failure rates can be described. A shape parameter b below 1 indicates an early failure behavior, while a shape parameter b about 1 gives a hint regarding a random failure. Runtime-related failure behaviors are often characterized by a shape parameter b above 1. If the failure rates of these three failure behaviors are plotted graphically, the bathtub curve results, cf. Figure 1.

When simple distribution models can no longer adequately represent the failure behavior due to high complexity or the presence of multiple failure causalities, mixed population approaches are used. A distinction is made between alternative and competing models. In alternative models, different functions of the different or same function type are combined. Competing models are used when different damage causalities occur at the same time (cf. Bracke, 2024).

In this paper, an alternative mixed population approach based on the two-parameter Weibull distribution is used, cf. (7). Here, three Weibull cumulative density functions with index i from one to three are combined using weights p_i, which ranges from zero to one depending on the number of failures in the specific failure phase in relation to the total quantity. The fitting of the parameters and weights is made using Maximum Likelihood Estimation via the EM Algorithm (Dempster et al., 1977).

$$F(x) = p_1 \times \left[1 - e^{-\left(\frac{x}{T_1}\right)^{b_1}}\right] + p_2 \times \left[1 - e^{-\left(\frac{x}{T_2}\right)^{b_2}}\right] + p_3 \times \left[1 - e^{-\left(\frac{x}{T_3}\right)^{b_3}}\right] \tag{7}$$

4. The Johnson System

The Johnson system is a family of distributions introduced by Norman Lloyd Johnson in 1949, that translate an observed, non-normal variate to one conforming to the standard normal distribution. Such distribution families are often used to summarize data in a function, as the different families can represent the data set in a flexible, adaptable way (Johnson, 1949). The three distribution families of the four-parametric Johnson system are represented by the form: $z = \gamma + \delta\, k_i(x, \lambda, \eta)$. With the selection of z as the standard normal variable and k_i (x, λ, η) the Johnson system represents as many forms as possible. In this way, the three distribution families can be modelled according to the following functions:

$$k_1(x; \lambda; \eta) = ln\left(\frac{x-\eta}{\lambda}\right) \tag{8}$$

$$k_2(x; \lambda; \eta) = ln\left(\frac{x-\eta}{\lambda + \eta - x}\right) \tag{9}$$

$$k_3(x; \lambda; \eta) = sinh^{-1}\left(\frac{x-\eta}{\lambda}\right) \tag{10}$$

If the parameter λ is replaced by $\gamma *= \gamma - \delta ln(\lambda)$, so that $z = \gamma * + \delta\ ln(x - \eta)$ applies, the SL-distribution (L = lognormal) results from k_1 in (8), which can therefore be modelled with three parameters. The k_2 in (9) form represents a system bounded distribution (Sb), which is bounded with $\eta, \eta + \lambda$. SU with k_3 in (10) enables an unbounded distribution representation. Thus, the lognormal curve represents a dividing line between the limited Sb region and the continuing SL distribution (Slifker and Shapiro, 1980).

4.1. System bounded (Sb Johnson)

The boundaries of the Sb Johnson are realized by two limit parameters $\eta\ and\ \lambda$, where η represents the left as a minimum and λ the right limit as a maximum of the analyzed data. So $\eta < x < \eta + \lambda$ for the left and for the right side the condition $\lambda > 0$ must be met. In addition, the following applies to the two shape parameters $\gamma\ and\ \delta$: $-\infty < \gamma < \infty$ and $\delta > 0$. In general, an increase in the absolute value of γ also increases the skewness, while an increase in δ increases the kurtosis. The probability density function of the four-parameter model of the Sb Johnson random variable x is given by the form according to (11):

$$f(x) = \frac{\delta \cdot \lambda}{\sqrt{2\pi} \cdot (\eta+\lambda-x) \cdot (x-\eta)} \cdot e^{(-\frac{1}{2}(\gamma+\eta \cdot ln(\frac{x-\eta}{\lambda-x+\eta}))^2} \tag{11}$$

where $0 < x < \lambda$, $\delta > 0$, $-\infty < \gamma < \infty$, $\lambda > 0$, and $\gamma + \delta \ln\,[x/(\lambda - x)] = z_x \approx N(0,1)$. The cumulative distribution function shown in (12) (Kudus et al., 1999):

$$F(x) = \Phi\left[\gamma + \delta \cdot ln\left(\frac{x}{\lambda - x + \eta}\right)\right] \tag{12}$$

To visualize different parameterizations of the probability density function (a) and the cumulative distribution function (b), different curves are shown in Figure 2.

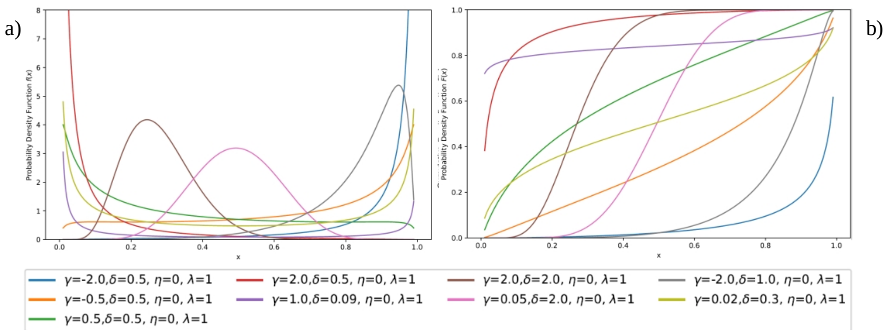

Fig. 2. Sb Johnson distribution characteristics with different parameter sets.
(a) Probability density function; (b) Cumulative distribution function.

4.2. Applications of Sb Johnson

In direct comparison to the Weibull distribution, various areas were identified in which the Sb Johnson distribution provides more precise results. For example, Kudus et al. described in 1999 that the system bounded distribution for diameter and height data is consistently better than the Weibull, beta, gamma and normal distributions due to the flexibility of the four parameters. Another application area is the modeling of offshore wind speeds. The Sb Johnson was here adapted to long-term time series of offshore wind farms that could be reliably predicted for various sea areas (Soukissian, 2013).

According to Cugerone and De Michele 2014, the limiting boundary parameters η and λ of the Sb Johnson can also be used for the statistical description of raindrop sizes, which are subjected to a physical size limit. In addition, the statistical moments are limited by the clear boundary values, which means that the two parameters γ and δ can represent the limited distribution range in many variations. Thus, in the described study, eight data sets from eight different locations and a one-minute sequence were fitted with the Sb Johnson, gamma and lognormal curves. The parameters were all determined using the maximum likelihood estimation. The comparison between the three models showed that the Sb Johnson curve best represents the size of the raindrops (Cugerone and De Michele, 2014).

However, in the examples mentioned it is also pointed out that problems often occur with Sb Johnson due to the limiting parameters when its parameters are determined using the maximum likelihood method. Due to the limitations, it is unlikely to converge to the end in every case, meaning that the fitting method has to be modified. To this end, J. F. Slifker and S. S. Shapiro developed in 1980 a corresponding strategy to systematically adapt the Sb Johnson to the required conditions so that it is more likely to converge.

In general, it can be summarized that the Sb Johnson system theoretically has particular advantages over other distribution models, if the distribution to be adjusted has limiting boundary conditions. Due to the flexibility provided by the two boundary parameters and the two shaping parameters, these limited distributions are well represented. Nevertheless, practical implementation poses challenges, contributing to the limited use of the Johnson system (Flynn, 2006).

4.3. Comparison Interpretation regarding Weibull and Sb Johnson

Comparing the three-parameter Weibull distribution to the four-parameter Johnson Sb distribution, the following differences arise: The Weibull distribution is often employed due to its straightforward interpretability of parameters. However, it encounters limitations when dealing with more complex distribution behaviors because of its three-parameter representation. In such cases, the Weibull distribution is adapted to the complexity through a mixed population approach, but this complicates interpretability. In contrast to the three-parameter Weibull, the Sb Johnson has one additional parameter, theoretically allowing the representation of more complex curve shapes. Consequently, only one additional parameter needs to be determined and interpreted.

A potentially comparable interpretation of parameters lies between the left threshold of Sb Johnson η and the threshold t_0 of Weibull. While t_0 describes the first failure of a damage mechanism, this can also be equated with the limiting property of the left boundary η. The parameter b describes both the skewness and kurtosis of the distribution, which in the Sb Johnson distribution are influenced by the two parameters γ and δ. In contrast to that, the characteristic lifespan T, also referred to as the expectancy value of the Weibull distribution, does not have comparable scaling properties to the right boundary λ of the Sb Johnson.

Overall, comparing the properties of the parameters proves to be challenging, and they should be analyzed explicitly for each specific application. For this reason, a case study is carried out in this article in order to compare the Johnson parameters with interpretable parameters of the Weibull distribution and to evaluate the applicability of the SB Johnson distribution to technically complex damage causalities.

5. Case study

The following section presents a case study to compare the parameters of the Sb Johnson distribution with those of the three-parameter Weibull distribution. First, various synthetically generated data sets are fitted and interpreted using the Sb Johnson distribution. In the second part, the three-parameter Weibull distribution is applied to the same data sets, followed by a comparison of the distribution parameters through a correlation analysis.

5.1. Base of operations: Data set

Three synthetic datasets are generated in total, each representing different failure behaviors. The diverse failure behaviors are synthesized based on the Weibull distribution. The first data set comprises the failure behavior over the entire product lifetime, characterized by early, random and runtime-related -failures. A segmentation of the data set according to (Bracke and Haller, 2009) shows the data structure according to an early failure behavior between a range of 1 and 135503 miles, while the random failures appear between 25091 and 138677 miles. The failure times of the runtime-related failures are between 53607 and 214279 miles. The combination of all failure behaviors enables the representation of the entire bathtub curve. In contrast, the second data set is based on three runtime-related failures, with ranges of 48418 – 282881, 327636 – 868966 and 1154177 – 2061610 thousand miles. In particular, to make the data basis for the correlation analysis more representative, a set of 30 runtime-related failure cases is considered in a third data set. The 30 synthesized data rows contain Weibull parameters with a slope b = 1.29 – 2.70; t_0= approx. 30000 miles; T = 140000 with 30 failure times each.

5.2. Sb Johnson analyses: Fitting and interpretation

In order to improve the interpretability of the Sb-Johnson parameters in the context of the failure mechanisms, the Sb-Johnson distribution is individually adapted to different failure mechanisms. The parameters are estimated using the standard maximum likelihood method from the scipy.stats library (Python) and fitted separately for the different failure mechanisms. As shown in Figure 3, the failure probability of the three failure segments of the first data set are shown on the left, while the corresponding failure rates are shown in the middle. On the right side, the three segments are merged and fitted together with a Sb Johnson distribution model ($F(x)$) and the corresponding cumulative frequency, cf. (4). All parameter results are listed in Table 1.

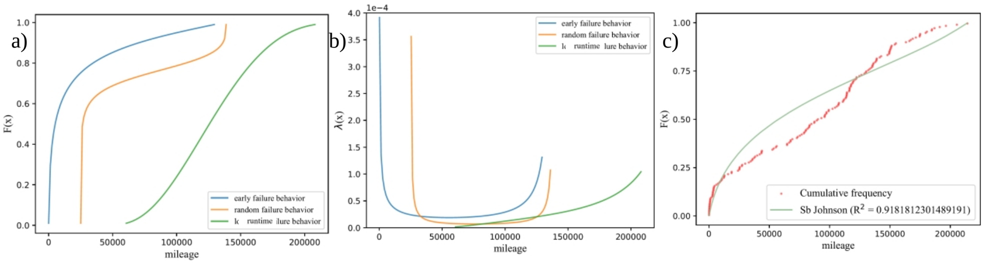

Fig. 3. Sb Johnson distribution models of the first dataset, separate consideration of the failure modes.
(a) Failure probability $F(x)$; (b) Failure rate $\lambda(x)$; (c) Failure probability $F(x)$ entire first dataset.

Analyzing the failure probability, it becomes clear that the Sb Johnson characterizes the three failure mechanisms differently. The early failures are characterized by a significant gradient at the beginning and a stagnating plateau at the end. Similarly, the random failures show a plateau phase in the middle and rise again towards the end of the distribution range. In contrast to that, a steady increase in the default can be seen in the runtime- failures. The start and end points of the different distributions also show how the limiting parameters η and λ affect the distributions. A comparison of the values in Table 1 clearly shows that the left limit of the random failure distribution is almost identical to the start of the distribution. This is also shown by the failure rate of early and random failures shown in the center of Figure 3. Both the beginning and the end of the distribution are marked by exponentially increasing slopes. Nevertheless, when the Sb Johnson is applied to fit the diverse failure of the entire first data set, a swift escalation is evident initially. However, it fails to encapsulate the subsequent plateau phase within the mid-range of failures and the escalating number of failures towards the end without deviations. Hence, the assumption is posited that the Sb Johnson does not achieve complete convergence and, consequently, does not align more closely with the data trend.

Similar to Figure 3, Figure 4 shows the second dataset failure probability on the left and the failure rate in the middle respectively. These represent the separately fitted failure behaviors of the second data set, which are summarized in Figure 4 on the right as one failure probability. The corresponding parameters are also shown in Table 1.

The failure probability on the left side shows a similar behavior to the runtime-related failures of the first data set. All failure mechanisms show a steady increase and are clearly bounded by the limit parameters in the respective distribution range. While the first fitted the runtime failure behavior exhibits a comparatively steep ascent, failure behavior models two and three show an exponential increase after a short initial phase. When fitting the three failure behaviors alongside a combined Sb Johnson distribution model (Figure 4 c), a broad trend is observed, similar to the first dataset. However, this method again fails to capture the specific failure curves, which leads to significant differences in the second plateau phase.

Table 1. Parameters of the fitted Sb Johnson by dataset and segment.

Dataset	Segment	Sample size n	Shape γ	Shape δ	Minimum η	Maximum λ
1	1	50	1,264780765	0,387735012	-2,3768	137743,9891
1	2	25	0,69126627	0,157921899	25091	113586,11
1	3	90	461898845	1,350494256	36389,7982	214758,2731
1	all segments	165	0,4410	0,439553895	126,0365	214971,4435
2	1	19	3,10986622	1,85048746	7461,61574	702727,667
2	2	41	-0,41221556	2,50482982	-71667,0542	1233554,7
2	3	80	-0,6611816	1,95825391	759501,669	1558965,14
2	all segments	140	-0,0664	0,52156445	38075,9702	2028611,949

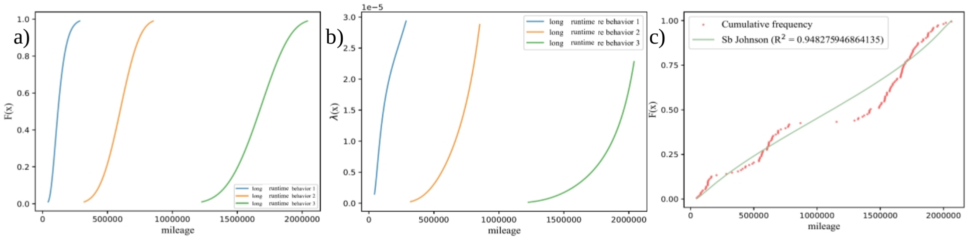

Fig. 4. Sb Johnson distribution models of the second dataset, separate consideration of the failure modes. (a) Failure probability $F(x)$; (b) failure rate $\lambda(x)$; (c) Failure probability $F(x)$ entire second dataset.

5.3. Weibull analyses: Fitting and interpretation

In this section, an analysis of the first and second dataset with Weibull distribution models is conducted to get a comparison base for the Sb Johnson interpretation. At first the different failure behaviors are analyzed individually using the segmented data sets. Secondly, a mixed population approach is applied for the two datasets considering all failure behaviors simultaneously.

5.3.1 Separate consideration of the failure modes

The first data set is separated into three parts: The first segment contains the early failure mode, the second contains the random failure mode and the third segment is characterized by the runtime-failure behavior. Three-parameter Weibull distribution models are fitted for each segment. In Figure 5, the failure probability of the three sections is shown on the left side and the failure rate is shown on the right side. The corresponding parameters are documented in Table 2.

Table 2. Parameters of the Weibull distribution models of the first dataset by segment.

Segment	Sample size n	Shape b	Scale T	Threshold t_0
1	50	0.547	11,459	0
2	25	1.043	60,704	23.323.60
3	90	2.790	138,623	36.893.28

The different characteristics of the three failure behaviors can be clearly seen. The curve of the failure probability of the early failure behavior increases significantly faster than the other failure probabilities. The failure rates of the three failure behaviors build the bathtub curve: The early failure behavior shows a decreasing failure rate, while the failure rate of the random failure behavior is nearly constant and the failure rate of the runtime-related failure behavior has an increasing trend.

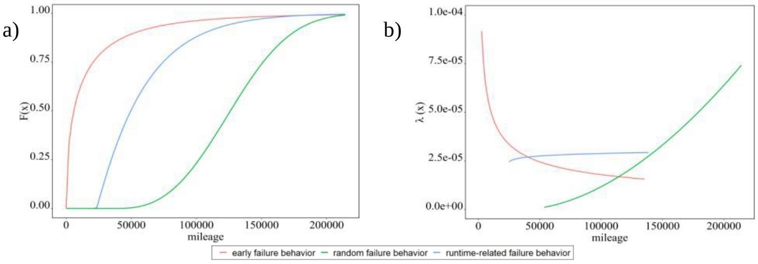

Fig. 5. Weibull distribution models of the first dataset, separate consideration of the failure modes.
(a) Failure probability $F(x)$; (b) Failure rate $\lambda(x)$.

In the second dataset, three different runtime-failure behaviors are separated. Three-parameter Weibull distribution models are fitted for each segment. In Figure 6, the failure probability of the three sections is shown on the left side and the failure rate is shown on the right side. The corresponding parameters are documented in Table 3.

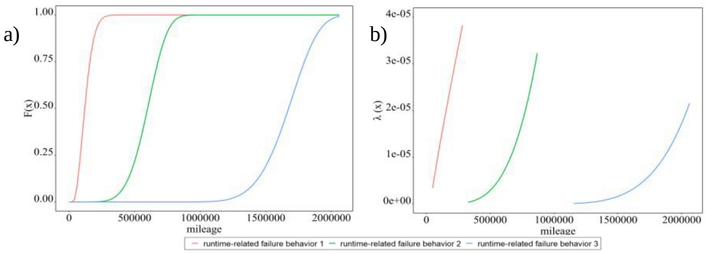

Fig. 6. Weibull distribution models of the second dataset, separate consideration of the failure modes.
(a) Failure probability $F(x)$; (b) Failure rate $\lambda(x)$.

Table 3. Parameters of the Weibull distribution models of the second dataset by segment.

Segment	Sample size n	Shape b	Scale T	Threshold t_0
1	20	1,908	138.976	31.115,62
2	40	4,646	639.381	107.098,88
3	80	5,571	1.735.631	764.163,49

The impact of an increasing shape parameter b is clearly visible: The larger the shape parameter, the faster the failure probability increases and the flatter the failure rate becomes.

5.3.2 Mixed population approach using Weibull analyses

A mixed population approach is used to consider all failure behaviors in one distribution as a comprehensive model. No segmentation is necessary. The mixture distribution used in this paper is based on the two-parameter Weibull distribution and contains three parts, cf. (7). In Figure 7, the failure probability of the Weibull Mixture model is compared with a three-parameter Weibull distribution fitted over the whole dataset. In addition, the Cumulative frequency, cf. (4), is shown. The coefficient of determination is given as the squared spearman

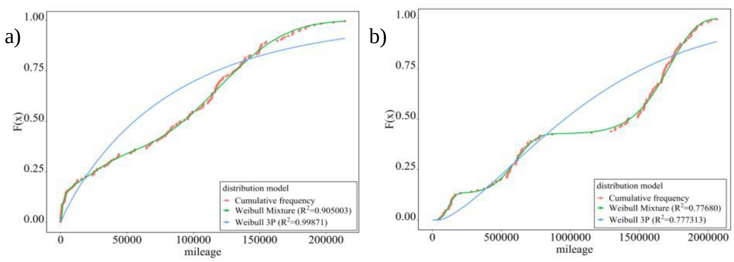

Fig. 7. Comparison of failure probabilities $F(x)$ of Weibull Mixture model and three-parameter Weibull distribution with Cumulative frequency; with coefficient of determination R^2. (a) First dataset; (b) Second dataset.

correlation coefficient of the Weibull model and the Cumulative frequency. The parameters of the fitted Weibull models are documented in Table 4.

Table 4. Weibull Mixture parameters by Segment and three-parameter Weibull model fitted for the whole dataset. First and second dataset.

Dataset	Segment	Percentage p	Sample size n	Shape b	Scale T	Threshold t_0
1	1	0.199	33	0.669	4,144	-
1	2	0.126	21	2.307	29,209	-
1	3	0.675	111	3.347	39,045	-
1	all segments	1	165	0.923	82,428	0
2	1	0.134	19	3.37	131,250	-
2	2	0.293	41	5.28	633,396	-
2	3	0.572	80	10.08	1,742,292	-
2	all segments	1	140	1.433	1,226,236	46,405.79

It can be clearly seen, that the Weibull Mixture distribution represents the complex failure behavior much better than the single three-parameter Weibull distribution for both datasets. The segments with their percentages are exactly found; this can be seen by comparing the parameters of the segments in Table 4 with the parameters in Tables 2 and 3. The breaks in the failure probabilities are accurately depicted by the mixture distribution model.

5.4. Discussion: application possibilities and limitations

In this section, the achieved results of the applied distribution models are discussed and compared based on the failure rate. Based on this discussion, the suitability of the Sb Johnson model for application to various failure mechanisms is evaluated. A correlation analysis is used for the interpretation of the determined Sb Johnson parameters in a direct comparison between Sb Johnson and the three parametric Weibull models.

Figure 8 shows the fitted failure rates of Sb Johnson, Weibull Mixture and three-parametric Weibull for the entire data sets. For both data sets, there is no adequate fit of the purely three-parametric Weibull model. In contrast, due to the complexity of the failure probability already discussed, it can be assumed that the Weibull mixture distribution provides a very accurate representation of the failure behavior. Upon close examination of the fitted Sb Johnson model, the left-hand failure rate curve exhibits a notable resemblance to the bathtub curve. Nevertheless, an assessment of the corresponding failure probabilities reveals that while a broad overall trend is discernible, the distinct failure phases are only indicated with a certain deviation. The same occurs to the three wear mechanisms depicted on the right, constituting the second dataset. It can be concluded that the Sb Johnson model with four parameters is capable of approximately replicating the bathtub curve. However, this is associated with certain deviations. The transitions between different failure behaviors, which can be described with the Weibull mixture distribution model, are not reproducible with the Sb Johnson distribution model. It is assumed that the number of parameters is the limiting factor here and a lack of convergence also impairs the accuracy at significant points.

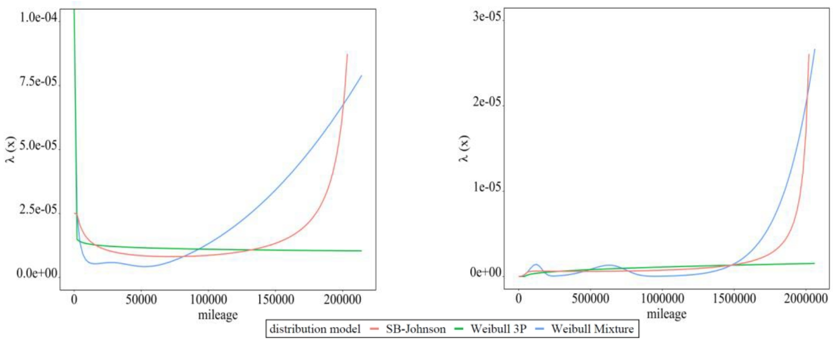

Fig. 8. Comparison between failure rates of Sb Johnson, Weibull Mixture and three-parametric Weibull fitted to whole first (a); second dataset (b).

To interpret the Sb Johnson parameters, a set of 30 run time failures (third dataset) is fitted by Sb Johnson as well as by three parametric Weibull distributions. The relation between the determined model parameters is analyzed with a correlation analysis according to Spearman. The corresponding correlation matrix can be found in Figure 9.

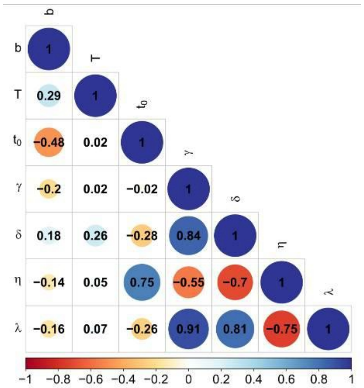

Fig. 9. Spearman correlation between three-parametric Weibull and Sb Johnson with the third dataset.

Overall, only very weak to weak correlations can be found between the model parameters of Weibull and Sb Johnson distribution. The one exception is the strong correlation of 0.75 between the Johnson parameter η, which represents the left distribution limit, and the shape parameter t_0 of the Weibull distribution. This confirms the previously assumed comparability of a distribution limit resulting from the first failure. Furthermore, there are no significant correlations between the two distribution models. While there are further stronger correlations between all Sb Johnson parameters among each other, the Weibull parameters show almost no correlation with each other.

6. Summary

In this comparative case study, the suitability of the Sb Johnson model for analyzing technically complex failure behaviors was thoroughly investigated. Multiple synthetic datasets were generated, and the Sb Johnson model was compared against fits of a three-parametric Weibull distribution. An essential aspect of this examination was the fact that the Sb Johnson model has one more parameter than the Weibull distribution, theoretically enabling its application to more complex relationships.

The fits were analyzed not only for individual failure behaviors but also holistically, considering temporal dependencies between different failure behaviors. The results demonstrated that the Sb Johnson model was capable of approximating the general trend of failure rates, particularly in terms of the characteristic bathtub curve. However, deviations were also observed, particularly in representing specific failure phases with a degree of uncertainty. Therefore, additional studies involving only two temporally linked runtime-related failure behaviors may be of interest to align the parametric set ratio of the mixed Weibull distribution for comparability.

Furthermore, a notable exception was the strong correlation between the left Johnson distribution limit parameter η and the shape parameter t_0 of the Weibull distribution. In addition, no further correlations between the distributions were found using correlation analysis.

References

Bracke, S. 2024. Reliability Engineering. Data analytics, modelling, risk prediction. Springer, Berlin Heidelberg.

Dempster, A. P., Laird, N. M., and Rubin, D. B. 1977. Maximum Likelihood from Incomplete Data via the EM Algorithm. Journal of the Royal Statistical Society: Series B (Methodological) 39(1), 1-22.

Slifker, J. F. and Shapiro, S.S. 1980. The Johnson System: Selection and Parameter Estimation 22, 239-246

Kudus, K.A.,Ahmad, M.I. and Lapongan, J. 1999. Nonlinear regression approach to estimating Johnson S_B parameters for diameter data

Cugerone, K. and De Michele, C. 2014. Johnson SB as general functional form for raindrop size distribution

Flynn, M. R. 2006. Fitting human exposure data with the Johnson SB distribution.

Johnson, N. L. 1949. Systems of Frequency Curves Generated by Methods of Translation. Biometrika 36, 149-176

Johnson, N. L. 1949. Bivariate Distributions Based on Simple Translation Systems. Biometrika 36(¾), 297-304

Soukissian, T. 2013. Use of multi-parameter distributions for offshore wind speed modeling: The Johnson S_B distribution

Weibull, W. 1951. A Statistical Distribution Function of Wide Applicability. ASME J. Appl. Mech. 18(3), 293-297.