Advances in Reliability, Safety and Security, Part 6 – Kołowrocki, Magryta-Mut (eds.)
© 2024 Polish Safety and Reliability Association, Gdynia, ISBN 978-83-68136-18-0 (printed), ISBN 978-83-68136-05-0 (electronic)

Advances in Reliability, Safety and Security

ESREL 2024
Monograph Book Series

Reliability Evaluation Of Multistate Flow Network Using Pre-Ordered Minimal Paths

Partha Chakrabarti, Neeraj Kumar Goyal

Indian Institute of Technology Kharagpur, Kharagpur, India

Abstract

Reliability analysis of real-world infrastructure systems such as communication networks, power grids, and supply chain networks poses a challenging computational problem. These systems can be modeled as Multistate Flow Networks (MFN) where components have multiple capacity states between fully operational and failed states. Evaluating the reliability of MFNs is NP-hard as the network size increases. In this study, we propose a novel pre-ordering minimal paths set approach before applying it to two well-known recursive sum of disjoint product algorithms available in the literature. We have applied our proposed method to a benchmark network to validate the robustness and correctness of our approach. Additionally, to test the scalability of our algorithm, we applied it to large networks. Our method significantly reduce the number of mathematical operations for the well-known reliability evaluation methods by 39% and 45%, respectively

Keywords: network reliability, minimal paths, multistate flow networks, sum of disjoint products

1. Introduction

Critical infrastructure systems such as transportation and logistics (Wu et al., 2008), power transmission systems (Y.-K. Lin & Huang, 2014), and telecommunication networks (Y.-K. Lin & Huang, 2014) serve as the backbone of modern society. Modelling them as flow networks allows for analyzing the efficient movement of resources, goods, or data from one node to another in these complex interconnected systems. However, in reality, the components of such networks can fail randomly, leading to disrupted flows. For example, a software bug triggered an outage in Gmail in 2020, disrupting global connectivity and workflows for millions of end users. Severe weather events have been responsible for cascading failures in electrical power transmission networks, leading to massive blackouts. A fire incident halted production at a manufacturing facility in Ericsson in 2022, affecting supply chain networks worldwide. An error during a network device replacement caused an AWS system failure in 2021, disrupting connectivity and operations for thousands of online platforms and smart devices worldwide. A botched network software update in 2022 by Rogers Telecommunications caused widespread cellular, Wi-Fi, and landline service disruptions, crippling point-of-sale devices, ATMs, smart city infrastructure, emergency services, and more throughout Canada. Grounding of the massive container ship Ever Given in 2021 completely blocked maritime traffic in the Suez Canal for six days, disrupting critical global shipping networks and inducing cascading road freight reliability issues by delaying thousands of vehicles and goods deliveries. These real-world examples illustrate that reliability analysis is critical across interconnected networks to ensure resilience against disruptions. Multistate flow network models are often used to analyze reliability by modelling components with probabilistic degraded capacities (J. -S Lin et al., 1995). However, as the network size grows, reliability analysis becomes computationally intractable, placing it within the realm of NP-hard problems.

Our research focuses on a two-terminal multistate reliability metric that defines the likelihood that a multistate flow network can facilitate a required demand flow between a specified source and a destination node while considering degraded component capacities arising due to failures. The objective of this study is to develop an algorithm for the reliability evaluation of multistate flow networks that is more computationally efficient than the current Sum of Disjoint Products (SDP) methods (Datta & Goyal, 2017, 2019; Zuo et al., 2007a). This paper introduces a new path set-based ordering approach to advance the state-of-the-art. The paper is structured as follows. Section 2 provides a background on reliability evaluation techniques for multistate flow networks. Building on relevant literature, Section 3 establishes the preliminary modelling foundations. The methodology section, Section 4, presents the proposed SDP algorithm in detail, including an illustrative example. Rigorous computational experiments on test networks are discussed in Section 5, quantitatively demonstrating the significant efficiency improvements our proposed method offers. Section 6 summarizes the conclusions and discusses directions for further enhancing the model as part of future work.

2. Related Literature

Network reliability evaluation has been an important research area for several decades. Earlier works focused on binary-state networks where components were either functioning or failed. Over the years, researchers have proposed various algorithms for finding minimal paths and cuts to calculate binary network reliability. Some notable algorithms include Dijkstra's algorithm, Floyd-Warshall algorithm, and the maximum flow algorithm (Dijkstra, 2022; Floyd, 1962; Warshall, 1962).

However, many real-world systems have components with multiple performance levels between perfect functioning and complete failure. Hence, the calculation of two-terminal reliability for multistate networks is an important problem that has received significant research attention. As discussed, two-terminal reliability at demand level d (denoted as R_d) can be exactly evaluated using methods like Inclusion-Exclusion if all d-minimal paths (d-MPs) are identified (W. C. Yeh, 2005).

Efficient identification of minimum path sets that can satisfy a specified demand level plays a pivotal role in the reliability analysis of multistate flow networks. Early works in this domain were limited to binary systems until Lin et al. proposed an efficient algorithm to identify all minimum path sets satisfying a particular demand (J. Lin et al., 1995). Their approach yielded fewer d-MP enumerations, especially for non-series-parallel networks, compared to the work proposed by (Janan, 1985). This linear programming model formed the foundation for many follow-up studies on mapping minimum path sets (Forghani-elahabad & Bonani, 2017; Y.-K. Lin, 2001; W.-C. Yeh, 2002). For example, Lin (Y.-K. Lin, 2001) adapted this to handle unreliable nodes and links. Alternatively, Yeh (W.-C. Yeh, 2002) developed an enhancement using cycle-checking to verify each potential solution's feasibility as a valid d-MP. Beyond these two models, researchers have developed approximate and specialized algorithms. Satitsatian and Kapur proposed an approximation using minimal improvement paths (Satitsatian & Kapur, 2006). Ramirez-Marquez et al. identified potential d-MPs based on information sharing between selected edges (Ramirez-Marquez et al., 2006). Bai et al. and Yeh focused on finding d-MPs for all possible demands (Bai et al., 2015; W.-C. Yeh, 2018).

Once the d-MPs are found for SDP evaluation, Aggarwal et al.'s algorithm (Aggarwal et al., 1975) introduced single variable inversion (SVI) for binary networks. Abraham (Abraham, 1979) reduced terms in Aggarwal's approach using Boolean algebra and path ordering. Locks (Locks, 1987) further streamlined Abraham's method by alphanumeric ordering of paths and disjoint terms, improving efficiency. For multistate networks, Zuo et al. proposed the recursive SDP algorithm (Zuo et al., 2007), while Yeh (W.-C. Yeh, 2015) introduced an improved SDP using simplification to enhance efficiency. Datta et al. proposed an improved SDP algorithm where they have introduced three new rules for identifying redundant d-MPs, checking for disjoint d-MPs and disjointing non-disjoint d-MPs (Datta & Goyal, 2023).

3. Preliminaries

Consider a multistate network denoted by $G = (V, E)$ with a source node s and a destination node t, where $V = \{s, n_1, n_2, \dots, n_n, t\}$ is the set of nodes, $E = \{e_1, e_2, \dots, e_m\}$ is the set of arc/edges and $W = (W_1, W_2, \dots, W_m)$ is the set of maximal capacity vector with W_i being the maximal capacity for a_i for $1 \leq i \leq \text{n} + m$. A state vector $x = (x_1, x_2, \dots, x_m)$ is the current capacity (i.e., state) of each network component a_i defined by x_i with non-negative integer values ranging from 0 to W_i. Such a network(G) is considered to satisfy the following assumptions further:

- The flow within the network (G) adheres to the flow-conservation principle, i.e., the total flow in and out of a node is equal except for the source and the destination node.
- All demands are exclusively transferred from the source node(s) to the destination node(t).
- The source and the destination node do not fail, i.e., they are perfectly reliable.
- The capacities of distinct arcs and nodes exhibit statistical independence.

- Each arc and node capacity (excluding the source and destination) is a random variable with integer values distributed according to a specified discrete probability distribution.

3.1. Notations

d	Demand from source to destination node
s, t	Source node, destination node
$G\ (V, E)$	A Multistate flow network with $V = \{s, n_1, n_2, \dots, n_n, t\}$ is the set of nodes & E= $\{e_1, e_2, \dots, e_l\}$ being the set of edges
W	Maximal capacity vector where $W = \{w_1, \dots, w_l, w_{l+1} \dots, w_{l+n}\}\ \forall\ w = (n + l)$ network components.
a_i	Capacity notation for all network components
P_i	Minimal path for $i= 1, 2, \dots, k$
W	Maximal capacity state of a component a_i
$W(P_k)$	The maximal capacity of a path is equal to the minimum of the capacities of its components.
R_d	Reliability of a network for demand (d)
φ_{min}	Set of minimal d-MPs
φ'_{min}	Set of minimal d-MPs after ordering
F	Set of feasible flow vectors $f_1, f_2, \dots, f_k$ for $j = 1,2, \dots, k$
x_i	Current capacity state of a component
L	Number of d-MPs

3.2. Acronyms

MFN	Multistate flow network
SDP	Sum of disjoint products
RSDP	Recursive sum of disjoint products
iSDP	Improved sum of disjoint products
MP	Minimal path
d- MP	d- Minimal path
GCF	Greatest common factors
DP	Disjoint product

Lemma1. φ_{min} is the set of d-MP/ lower boundary points for demand (d).
Proof: Suppose $X \in \varphi_{min}$ with $F(X) \geq d$, but it is not a lower boundary point for d. This implies the existence of a point Y such that $Y < X$ and $F(Y) > d$. Now, since $Y < X$, we must also have $Y \in \varphi_{min}$ ($X \in \varphi_{min}$ and Y is a boundary point). However, this contradicts the assumption that X is a lower boundary point because there exists a Y with a higher function value, $F(Y) > d$, and Y is still in φ_{min} . Hence, X must be a lower boundary point for d. Conversely, let us assume X is a lower boundary point for d, i.e., $X \in \varphi$ but $X \in \varphi_{min}$. It implies that there exists a point $X \in \varphi$ such that $Y < X$. Now, if X is indeed a lower boundary point for d, we must have $F(X) \geq d$ because if $F(Y) < d$, it would contradict the definition of X as a lower boundary point. However, we assumed $Y < X$, which means $F(Y) > F(X)$. So, we have $F(Y) > F(X) \geq d$, which contradicts the assumption that X is a lower boundary point for d. Therefore, it must be the case that $X \in \varphi\_min$.

Theorem 1. Network reliability from the disjointed set of DPs can be calculated as given in Eq. (1).

$$R_d = P(\{X|X_1 \leq X\} \cup \{X|X_2 \leq X\} \cup \dots \cup \{X|X_L \leq X\}) = P(\cup_{i=1}^{L} X_i) \tag{1}$$

4. Proposed Approach

Let $P_1, P_2, \dots, P_k$ be the minimal path sets from source to destination obtained by applying the path generation algorithm proposed by Chaturvedi & Mishra (Chaturvedi & Misra, 2002). Here, we have introduced a novel ordering technique for the d-MPs. Suppose we have three d-MPs as $P_1 = (3, 2, 1,1,3)$, $P_2 = (2, 2, 0,1,1)$ and $P_3 = (4, 4, 3,3,2)$. Each of the component (a_i) for all the d-MPs to have a maximum capacity $W_i = 4$. We will order the d-MPs in a descending order starting from the MP, which can cover the maximum number of component capacity states starting from the current component capacity state (x_i).

The proposed ordering method prioritizes generating lower-order path sets before higher-order ones. For path sets of the same cardinality, priority is given to the path sets that share more common capacity states with previously generated sets. This ordering allows earlier elimination of overlapping states between path sets. Disjointing lower order sets first removes more shared capacity states per operation than disjointing with a higher order path set later.

If a higher order path set is disjointed with a lower order path set, fewer shared states are eliminated per operation since the higher order set contains a smaller number of shared capacity states between them. Delaying the removal of overlapping states increases the overall computational burden for reliability calculation.

For example, the capacity state of the components for P_1 ranges from $a_1 = [3-4], a_2 = [2-4], a_3 = [1-4], a_4 = [1-4], a_5 = [3-4]$, thus having a total number of 15 component capacity states. Similarly, P_2 will have 19 capacity states and P_3 will have 9 capacity states to cover all the components starting from their current state. After ordering the path sets $\varphi_{min} = \{P_2, P_1, P_3\}$.

Algorithm 1: Ordering minimal path sets

Input: Path set $\varphi_{min} = \{P_1, P_2, \dots, P_k\}$
Output: Ordered path set φ'_{min}

STEP 1: n← number of components
STEP 2: UB← $(W_1, W_2, \dots, W_n)$ /* Maximum capacity values for all components in P_k */
STEP 3: $\varphi min'$← $UB - \varphi_{min} + 1$ /* This step calculates the no. of capacity states each component can attain starting from its current state */
STEP 4: States ← $\mathrm{product}(\varphi_{min}{}^{T})$ /* Calculates the total no. of capacity states for all the components in a d-MP (P_k) */
STEP 5: [order, index] = sort (States, 'descend') /* Returns the index value in descending order for all the path sets starting from the path set having highest number of component capacity state
STEP 6: **FOR** $j \leftarrow 1\ to\ size(index, 2)$
$\varphi'_{min}(j,:) = \varphi_{min}(index(j),:)$
END FOR
/* Order the path sets in descending order based on the index value obtained in the previous step*/

4.1. System Reliability Evaluation

This section illustrates the steps in generating minimal path sets to the final reliability evaluation using RSDP (Zuo et al., 2007)

Step 1: Obtaining feasible and valid flows from minimal paths.

The feasible flow vectors $F = (f_1, f_2, \dots, f_k)$ are formed by allocating portions of the total demand d to the minimal paths $P_1, P_2, \dots, P_k$ such that:

- The flow allocated to each path P_i for $i = 1,2, \dots, k$ is nonnegative.
- The sum of flows allocated to all paths $\sum_{j=1}^{k} f_j = d$, where f_j is the flow allocated to the path P_k.

Several feasible flow vectors share common capacity states among their components, rendering them unable to accommodate the demand simultaneously. For example, a $F = (f_1, f_2, f_3, f_4) = (4, 3, 0, 0)$ having four MPs carrying a total demand of 7 units. Suppose a component a_i is common to both minimal paths P_1 and P_2 and the maximal capacity of the component a_i is 4 units, then $F = (f_1, f_2, f_3, f_4) = (4, 3, 0, 0)$ is not a feasible flow vector. We retain only the valid flow vectors from the set of feasible flow vectors, discarding infeasible ones.

Step 2: Transform each valid flow vector into boundary flow vectors.

The boundary flow vector corresponding to the valid flow vector is the component capacity state in which exactly the allocated demands of all the paths in the valid flow can pass from the source node(s) to destination node(t). The component's capacity states and probabilities are the key elements to evaluate the network reliability; therefore, valid flow vectors are converted into boundary flow vectors.

Step 3: Transform boundary flow vectors into lower boundary flow vectors or d-MPs.

Obtain the set of lower boundary flow vectors or d-MPs using Lemma 1.

Step 4: Ordering d-MPs

Order the d-MPs using Algorithm 1.

Step 5: Evaluate point reliability from ordered d-MPs

We calculate reliability by evaluating the probability of the union of events where the state vector is greater than or equal to at least one of the d-MPs $(X_1, X_2, \dots, X_L)$that have been identified. The final reliability expression can be formulated as given in Eq. (1).

4.2. An Illustrative Example

To illustrate the MFN distribution algorithm, we employ a simple supply chain network, as depicted in Figure 1, taken from relevant literature. This network comprises one supplier (S), two transfer centers(TC_1, TC_2) and one market(M) with six distinct routes$(a_1 - a_6)$. To reduce the calculation burden of the RSDP algorithm, we use the cumulative probability of the capacity states of different routes, as given in Table 1. The supervisor would like to know the reliability of the MFN to pass 7 units of demand/goods simultaneously from supplier (S) to market (M) $(i.e. R_7)$.

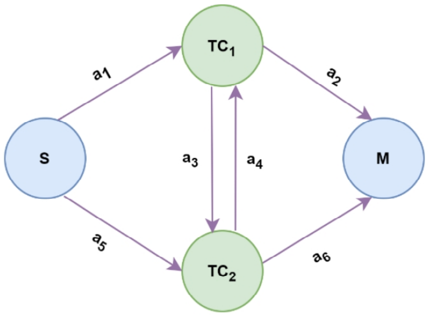

Fig. 1. A benchmark supply chain network.

Table 1. The Cumulative Probabilities of component capacities in Fig. 1.

Component	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Capacity 0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Capacity 1	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Capacity 2	0.97	0.97	0.97	0.97	0.97	0.97	0.98	0.98
Capacity 3	0.95	0.95	0.95	0.95	0.95	0.95	0.96	0.96
Capacity 4	0.90	0.90	0.90	0.90	0.90	0.90	0.94	0.94
Capacity 5							0.92	0.92
Capacity 6							0.90	0.90

The minimal path sets of the network between the source node (S) and the destination node (M), as determined by the algorithm (Chaturvedi & Misra, 2002) are listed in Table 2. There exist four minimal paths: P_1, P_2, P_3, and P_4. Let TC_1 and TC_2 be a_7 and a_8. Path P_2 comprises components a_1, a_7 a_3, a_8 and a_6, with maximum capacities of 4, 6, 4, 6, and 4, respectively. Hence, the maximum demand that P_1 can accommodate is the minimum among $\{4, 6, 4, 6, 4\}$, which equals 4 units. Similarly, each of the minimal paths P_2, P_3 and P_4 can transport a maximum of 4 units of demand from node S to node M.

Table 2. Minimal path sets for the network in Fig. 1.

Path Number	Path sets	Max. Path capacity
P_1	$a_1 a_7 a_2$	4
P_2	$a_1 a_7 a_3 a_8 a_6$	4
P_3	$a_5 a_8 a_4 a_7 a_2$	4
P_4	$a_5 a_8 a_6$	4

<u>Step 1: Obtaining Feasible and Valid Feasible Flows from Minimal Paths</u>

After identifying the minimal paths, we generate 80 feasible flow vectors. From these 80 feasible flow vectors, we retain only the valid ones and discard those that are not feasible. The valid flow vectors are presented in Table 3.

Table 3. Valid flow vectors obtained from minimal paths.

Valid Flows vs Paths	P_1	P_2	P_3	P_4	Valid Flows vs Paths	P_1	P_2	P_3	P_4
F_1	1	1	2	3	F_7	2	3	1	1
F_2	1	1	3	2	F_8	3	2	1	1
F_3	1	2	2	2	F_9	3	3	0	1
F_4	2	1	2	2	F_{10}	3	3	1	0
F_5	2	2	1	2	F_{11}	3	4	0	0
F_6	2	2	2	1	F_{12}	4	3	0	0

Step 2: Transform each valid flow vector into boundary flow vectors.

After obtaining the valid flow vectors, we convert them into boundary flow vectors, as given in Table 4, representing the current state of all the components.

Table 4. Valid flow vectors obtained from minimal paths.

X_k	Component capacities								X_k	Component Capacities							
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
X_1	4	3	3	2	3	4	6	6	X_7	4	4	1	1	3	3	4	5
X_2	3	4	2	3	4	3	6	6	X_8	3	3	1	1	4	4	5	4
X_3	4	4	2	2	3	3	5	6	X_9	4	3	1	0	3	4	4	4
X_4	3	3	2	2	4	4	6	5	X_{10}	3	4	0	1	4	3	4	4
X_5	4	3	2	1	3	4	5	5	X_{11}	4	4	0	0	3	3	3	4
X_6	3	4	1	2	4	3	5	5	X_{12}	3	3	0	0	4	4	4	3

Step 3: Transform each valid flow vector into boundary flow vectors.

We obtain the set of lower boundary flow vectors or d-MPs listed in Table 5 using Lemma 1.

Table 5. Lower boundary flow vectors/ d-MPs.

X_k	Component Capacities							
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
X_1	4	3	1	0	3	4	4	4
X_2	3	4	0	1	4	3	4	4
X_3	4	4	0	0	3	3	3	4
X_4	3	3	0	0	4	4	4	3

Step 4: Ordering d-MPs

After obtaining the lower boundary flow vectors, we order them using Algorithm 1. The ordered set of d-MPs are as follows:

$X_1^o = \{4,4,0,0,3,3,3,4\}$; $X_2^o = \{3,3,0,0,4,4,4,3\}$; $X_3^o = \{4,3,1,0,3,4,4,4\}$; $X_4^o = \{3,4,0,1,4,3,4,4\}$

Step 5: Evaluate point reliability from ordered d-MPs

In this step, we will assess the point reliability for the network depicted in Figure 1 using the RSDP algorithm as stated in Theorem 1 using the cumulative probabilities of components given in Table 1, as follows:

$R_7 = P(X|X_7 \leq X)$ for a 7-MP X_7 = 0.7431667542

5. Results and Discussions

RSDP (Zuo et al., 2007) stands out as the most widely used and straightforward sum of disjoint products algorithm for evaluating the reliability of multistate flow networks. Yeh simplified the RSDP algorithm by leveraging the Greatest Common Factor (GCF) and Property 9 (W.-C. Yeh, 2015) to reduce the multiplications required in the reliability evaluation process. We will compare RSDP and iSDP with our proposed method, which involves ordering path sets, to assess the degree of improvement our algorithm offers. To ensure independence from specific computer hardware or software, we will quantify the total number of summations and multiplications needed to compute the reliability for each disjoint product. The total number multiplications to calculate R_7 with and without ordering path sets for RSDP and iSDP is listed in Table 6.

Table 6. Number of calculations to calculate R_7.

SDP Method	Without path set ordering	Proposed (With path set ordering)
RSDP	105	77
ISDP	64	24

As the number of d-MPs increases the computational power required to calculate reliability grows exponentially resulting to NP-hard problem. In order to test the practicability and efficiency of our proposed method, we generate a dataset containing 400 MPs. All models were implemented using MATLAB and executed on a computer equipped with an INTEL(R) Core (TM) i5-7500 CPU running at 3.40GHz, with 4 GB of RAM. Table 7 lists the total number of calculations required to evaluate the point reliability for the given set of MPs without path set ordering, and our proposed methodology (with path set ordering). On average, our proposed methodology of ordering the path sets results in a 39% decrease in the number of multiplications for RSDP and a 45% decrease in the number of multiplications for iSDP. However, there is only a 16% decrease in the summations. Figure 2 and Figure 3 show the computational time required to calculate reliability using sets of d-MPs. It is observed that the time required to calculate reliability for a set of 400 d-MPs using RSDP and iSDP is 12.43 seconds and 7.11 seconds, respectively. Using our proposed methodology, we can calculate reliability for the same 400 MP set in 6.81 seconds with RSDP and 4.78 seconds with iSDP. Thus, our proposed methodology reduces the computational effort for reliability evaluation by around 50% compared to standard RSDP and iSDP approaches.

Table 7. Number of calculations without path set ordering and with path set ordering.

d-MPs	Without path set ordering			Proposed (with path set ordering)		
	No. of summations	No. of multiplications		No. of summations	No. of multiplications	
	RSDP/ ISDP	RSDP	ISDP	RSDP/ ISDP	RSDP	ISDP
40	1137	7138	5769	1042	5722	3967
80	4472	25487	15121	3837	14857	8035
120	10287	58871	33652	8355	25869	14919
160	19389	86255	52449	15157	48645	22444
200	28033	145755	82453	22858	59317	24736
240	38686	179420	107863	34194	116018	40255
280	51519	224797	135865	47193	154419	97019
320	66180	274427	177672	56301	197659	104730
360	72370	321657	189762	59169	205629	131383
400	100042	366305	211576	79981	235714	154217

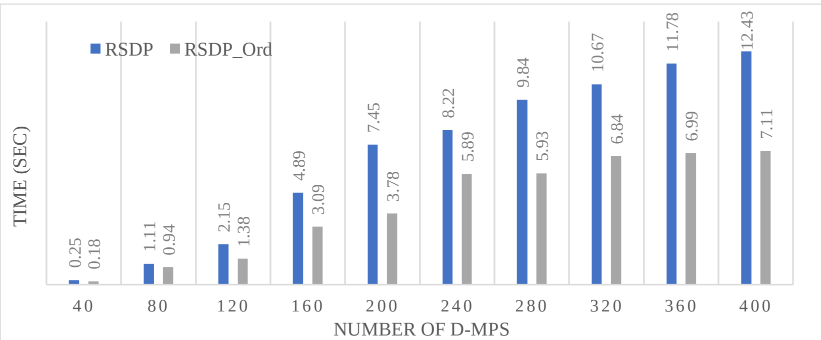

Fig. 2. Computational time for RSDP vs RSDP ordered.

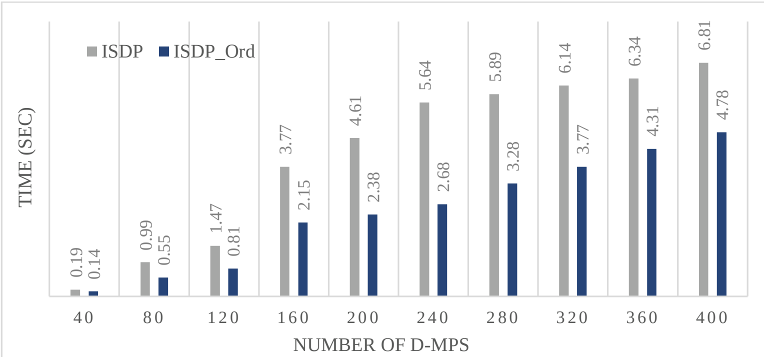

Fig. 3. Computational time for iSDP vs iSDP ordered.

6. Conclusion and future direction

Reliability evaluation of networks is critical for infrastructure systems such as communication networks, power grids, and supply chains. As these networks grow in size and complexity, efficiently analyzing their reliability becomes increasingly important. In this work, we have proposed a novel method of pre-ordering minimal paths sets prior to applying benchmark reliability evaluation algorithms like RSDP and iSDP. Our results demonstrate that this pre-processing step can significantly reduce the computational effort required by these algorithms. By improving the scalability of reliability analysis, our method enables more accurate and timely evaluation of large multistate flow networks. This has important implications for monitoring and maintaining critical infrastructure systems. The reduced computational burden also facilitates more frequent assessment of network reliability and probabilistic failure analysis. In conclusion, pre-ordering of minimal paths sets is an effective strategy for enhancing the capabilities of standard reliability evaluation techniques.

While this work has focused on efficient two-terminal reliability evaluation, real-world networks often involve multiple source and destination nodes. This leads to the multi-terminal reliability problem, which warrants further research. Our proposed approach of pre-ordering minimal paths sets could be extended to analyze multi-terminal reliability. Additionally, many networks support multi-commodity flows between various source-destination pairs. Incorporating such multi-commodity flows into the reliability analysis introduces further complexity. As future work, the scalability and computational gains of our proposed method could be assessed in the context of multi-terminal, multi-commodity reliability evaluation.

References

Abraham, J. A. 1979. An improved algorithm for network reliability. IEEE Transactions on Reliability, 28(1), 58–61.

Aggarwal, K. K., Misra, K. B., & Gupta, J. S. 1975. A fast algorithm for reliability evaluation. IEEE Transactions on Reliability, 24(1), 83–85.

Bai, G., Zuo, M. J., & Tian, Z. 2015. Search for all d-MPs for all d levels in multistate two-terminal networks. Reliability Engineering & System Safety, 142, 300–309.

Chaturvedi, S. K., & Misra, K. B. 2002. An efficient multi-variable inversion algorithm for reliability evaluation of complex systems using path sets. International Journal of Reliability, Quality and Safety Engineering, 9(03), 237–259.

Datta, E., & Goyal, N. 2023. An efficient sum of disjoint product method for reliability evaluation of stochastic flow networks using d-MPs. International Journal of System Assurance Engineering and Management, 1–19.

Datta, E., & Goyal, N. K. 2017. Sum of disjoint product approach for reliability evaluation of stochastic flow networks. International Journal of System Assurance Engineering and Management, 8, 1734–1749.

Datta, E., & Goyal, N. K. 2019. Evaluation of stochastic flow networks susceptible to demand requirements between multiple sources and multiple destinations. International Journal of System Assurance Engineering and Management, 10, 1302–1327.

Dijkstra, E. W. 2022. A note on two problems in connexion with graphs. In Edsger Wybe Dijkstra: His Life, Work, and Legacy (pp. 287–290).

Floyd, R. W. 1962. Algorithm 97: shortest path. Communications of the ACM, 5(6), 345.

Forghani-elahabad, M., & Bonani, L. H. 2017. Finding all the lower boundary points in a multistate two-terminal network. IEEE Transactions on Reliability, 66(3), 677–688.

Janan, X. (1985). On multistate system analysis. IEEE Transactions on Reliability, 34(4), 329–337.

Lin, J., Jane, C., & Yuan, J. 1995. On reliability evaluation of a capacitated-flow network in terms of minimal pathsets. Networks, 25(3), 131–138.

Lin, J. -S, Jane, C. -C, & Yuan, J. 1995. On reliability evaluation of a capacitated-flow network in terms of minimal pathsets. Networks, 25(3), 131–138.

Lin, Y.-K. 2001. A simple algorithm for reliability evaluation of a stochastic-flow network with node failure. Computers & Operations Research, 28(13), 1277–1285.

Lin, Y.-K., & Huang, C.-F. 2014. System reliability of assured accuracy rate for multi-state computer networks from service level agreements viewpoint. Journal of Systems Science and Systems Engineering, 23(2), 196–211.

Locks, M. O. 1987. A minimizing algorithm for sum of disjoint products. IEEE Transactions on Reliability, 36(4), 445–453.

Ramirez-Marquez, J. E., Coit, D. W., & Tortorella, M. 2006. A generalized multistate-based path vector approach to multistate two-terminal reliability. IIE Transactions, 38(6), 477–488.

Satitsatian, S., & Kapur, K. C. 2006. An algorithm for lower reliability bounds of multistate two-terminal networks. IEEE Transactions on Reliability, 55(2), 199–206.

Warshall, S. 1962. A theorem on boolean matrices. Journal of the ACM (JACM), 9(1), 11–12.

Wu, W., Ning, A., & Ning, X. 2008. Evaluation of the reliability of transport networks based on the stochastic flow of moving objects. Reliability Engineering & System Safety, 93(6), 838–844.

Yeh, W. C. 2005. A novel method for the network reliability in terms of capacitated-minimum-paths without knowing minimum-paths in advance. Journal of the Operational Research Society, 56(10), 1235–1240.

Yeh, W.-C. 2002. A simple method to verify all d-minimal path candidates of a limited-flow network and its reliability. The International Journal of Advanced Manufacturing Technology, 20, 77–81.

Yeh, W.-C. 2015. An improved sum-of-disjoint-products technique for symbolic multi-state flow network reliability. IEEE Transactions on Reliability, 64(4), 1185–1193.

Yeh, W.-C. 2018. Fast Algorithm for Searching d-MPs for all Possible d. IEEE Transactions on Reliability, 67(1), 308–315.

Zuo, M. J., Tian, Z., & Huang, H.-Z. 2007. An efficient method for reliability evaluation of multistate networks given all minimal path vectors. IIE Transactions, 39(8), 811–817.