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Genetic Algorithm And Availability Importance Measure Approach For Solving Redundancy Allocation Problem

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Abstract

The paper considers the redundancy allocation problem (RAP) in three fundamental system reliability structures: series, series-parallel, complex bridge. The system is composed of binary, heterogeneous, repairable components that are active in cold standby mode. Maximization of system availability under a cost constraint was used as the objective function. A continuous-time Markov model was developed to determine the availability of the system and subsystems. Two approaches were proposed to solve the RAP: a genetic algorithm and an iterative algorithm based on an availability importance measure (AIM). The results of the numerical examples indicated the greater efficiency of the genetic algorithm, however, the AIM-based algorithm needed a much shorter time to allocate standby components.

Keywords: redundancy allocation problem, Markov processes, genetic algorithm, availability importance measure, cold standby

1. Introduction

In real-world engineering systems, reliability and availability are critical to the efficient and safe execution of many processes. Redundancy in a system may refer to functional aspects (the system is equipped with additional over and above the necessary functions), parametric aspects (the system has increased values of technical parameters) and structural aspects (the system contains additional components) (Sharifi et al., 2021; Tarełko, 2017). Increasing the reliability structure by adding standby components reduces the risk of system failure and increases the safety of system operation. However, the design of redundant technical systems is almost always limited by the amount of available financial resources (Lins and Droguett, 2011). In the literature, this concern is referred to as the redundancy allocation problem (RAP).

Due to the wide variety of system structures and component types that exist, the optimization of redundancy presents a complex field of study (Zaretalab et al., 2020). The scientific literature is dominated by the division of components according to the space of possible states (binary and multi-state), homogeneity (homogeneous and heterogeneous), repairability (non-repairable and repairable), intensity of damage and repair over time (constant and time-dependent), and standby mode (cold, warm and hot) (Sharifi et al., 2019; Sharifi and Taghipour, 2022). Optimizing the placement of standby components in the system reliability structure is carried out according to an objective function that addresses at least one of three aspects: maximizing reliability (Yeh et al., 2021), maximizing availability (Zaretalab et al., 2022), and minimizing cost (Li and Zhang, 2022).

One of the most common approaches to RAP are evolutionary meta-heuristic algorithms (EAs), particularly genetic algorithms (GAs) (Keshavarz Ghorabaee et al., 2015). Maximizing system reliability with a cost constraint was carried out in the paper (Gholinezhad & Zeinal Hamadani, 2017) using a proprietary genetic algorithm. For a weighted k-out-of-n Khorshidi et al. (2016) used GA to maximize the system availability. A novel pseudo-parallel GA proposed by (Zhang et al., 2023) to optimize redundancy, in k-out-of-n systems with mixed redundancy. They used continuous-time Markov chains to assess system reliability. The developed algorithm was validated on four benchmarks. The paper (Tannous et al., 2011) compares the genetic algorithm with integer programming, indicating that GA can achieve better objective function results, however, in much longer computation time.

The high efficiency of evolutionary algorithms is the reason for their usefulness in solving the RAP, however, the relatively long computation time creates the need to look for alternative approaches. This article addresses these concerns by proposing an availability importance measure approach that was developed and comparing it with the genetic algorithm. To evaluate the performance of the two developed algorithms, a study was conducted on three benchmark systems: series, series-parallel and complex bridge.

2. Methodology

2.1. Assumption

In the analyzed redundancy allocation problem, the following assumptions have been adopted:

- The system consists of heterogeneous components;
- There are three possible states of component: working, cold standby and failure;
- Failures and repairs are independent Poisson processes;
- The failure and repair rates of components are constant;
- The costs of components are constant;
- At time $t = 0$ all components are operational;
- It is assumed that the switching time from the standby to the working state is equal to 0 (perfect switch).

2.2. Problem formulation

The paper considers the redundancy allocation problem with a single-criteria objective function, represented as maximizing system availability, according to the Equation (1):

$$Maximize\ A_S. \quad (1)$$

The main constraint condition is the total cost of the system, expressed as the sum of the costs of all its components:

$$\sum_{i=1}^{m} c_i n_i \leq C, \quad (2)$$

while satisfying the conditions of positive integer number of components and positive value of unit cost of each component (3) – (4):

$$n_i \in \aleph_+ for\ 1 \leq i \leq m, \quad (3)$$

$$c_i > 0\ for\ 1 \leq i \leq m. \quad (4)$$

According to the assumptions made, failure and repair rates are constant over time (5) – (6):

$$\lambda_i = const. \quad (5)$$

$$\mu_i = const. \quad (6)$$

2.3. Markov model

Continuous time Markov chains are utilized to describe multi-state technical systems that are characterized by exponential distributions of failure and repair processes (Oszczypała et al., 2023; Peiravi et al., 2020). One active component supported by $n - 1$ cold standby components can be presented as a 1-out-of-*n* subsystem. The active component is getting damaged with constant lambda intensity, while the other efficient components in cold standby mode cannot get failed. Each failed component is repaired independently with constant intensity μ. Thus, if a subsystem is in the state $S = k$, where $k \in [0,\ n]$ then it can transition to the state $S = k + 1$ with an intensity equal to $(n - k)\mu$ and to the state $S = k - 1$ with an intensity equal to λ. The graph of interstate transitions of the $n + 1$ state Markov model for the 1-out-of-*n* system is presented in Fig. 1.

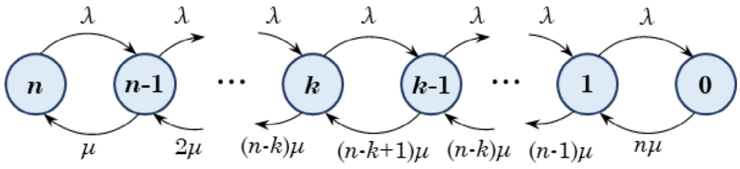

Fig. 1. Markov model transition graph for a 1-out-of-*n* system with cold standby components.

The Markov process of transition of reliability states is described by the transition intensity matrix $\boldsymbol{\Lambda}$(Ziółkowski et al., 2021; Żurek et al., 2020), according to the Equation (7):

$$\boldsymbol{\Lambda} = \begin{pmatrix} -n\mu & n\mu & 0 & \cdots & 0 & 0 & 0 \\ \lambda & -(\lambda+(n-1)\mu) & (n-1)\mu & \cdots & 0 & 0 & 0 \\ 0 & \lambda & -(\lambda+(n-2)\mu) & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -(\lambda+2\mu) & 2\mu & 0 \\ 0 & 0 & 0 & \cdots & \lambda & -(\lambda+\mu) & \mu \\ 0 & 0 & 0 & \cdots & 0 & \lambda & -\lambda \end{pmatrix}. \tag{7}$$

For a homogeneous stationary Markov process, the matrix equation (8) is satisfied:

$$\boldsymbol{\Pi} \cdot \boldsymbol{\Lambda} = \mathbf{0}. \tag{8}$$

The ergodic probabilities of the model are determined by assuming the following normalization condition (9):

$$\sum_{i=0}^{n} \pi_i = 1. \tag{9}$$

In the $n+1$ state space of the Markov model for a 1-out-of-*n* subsystem, the set $\boldsymbol{W}$ of availability states is equal to $\boldsymbol{W} = \{S_j : j = 1, 2, \dots, n\}$ where S_k means that there are k operational components in the subsystem. For the notation thus adopted, the availability of the system is expressed as the sum of the ergodic probabilities of all states belonging to the set $\boldsymbol{W}$:

$$A_i = \sum_{j:S_j \in W} \pi_j = \sum_{j=1}^{n} \pi_j. \tag{10}$$

2.4. Availability of system (series, series-parallel, bridge)

In this study, three main reliability structure benchmarks are considered for a series system, a series-parallel system and a complex bridge system. In the literature, these benchmarks are widely used to validate many optimization methods and algorithms (Kanagaraj et al., 2013; Mahdavi-Nasab et al., 2022; Ouyang et al., 2019). The structures of the systems are presented in Fig 2. An availability is a metric used to determine the reliable functioning of a repairable system. It is an objective evaluation that provides crucial information on the system's ability to operate without failures over time (Żurek et al., 2017).

In a series system, failure of one component (subsystem) causes failure of the entire system. Thus, the availability of a system is calculated as the product of the availability A_i of all its components (subsystems). For the system illustrated in Fig. 2a, A_S availability follows the equation (11):

$$A_S = \prod_{i=1}^{5} A_i. \tag{11}$$

The availability of the series-parallel system (Fig. 2b) is calculated according to the formula (12):

$$A_S = 1 - (1 - A_1A_2)(1 - (A_3 + A_4 - A_3A_4)A_5). \tag{12}$$

The availability of the complex bridge system (Fig. 2c) is determined by the equation (13):

$$A_S = A_1A_2 + A_3A_4 + A_1A_4A_5 + A_2A_3A_5 - A_1A_2A_3A_4 - A_1A_2A_3A_5 - A_1A_2A_4A_5 - A_1A_3A_4A_5 - A_2A_3A_4A_5 + 2A_1A_2A_3A_4A_5. \tag{13}$$

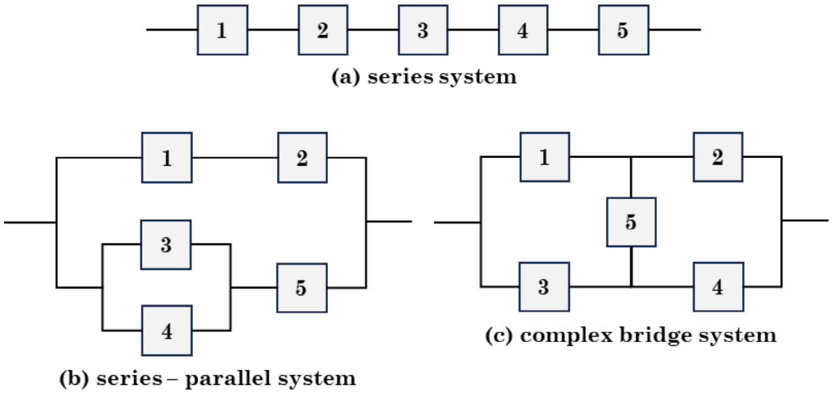

Fig. 2. Three main benchmarks from the literature (Hsieh, 2021): (a) series system, (b) series-parallel system, (c) complex bridge system.

2.5. Genetic algorithm

Genetic algorithms (GA) are a mathematical interpretation of the biological mechanism of evolution and belong to the group of meta-heuristic methods. The algorithm is based on three fundamental processes, i.e. selection, crossover and mutation. The first step of GA is to randomize the initial generation of solutions. The chromosomes that encode the distribution of components within the system are subsequently decoded. Based on the decoded chromosomes, the availability of subsystems is evaluated using Markov models and then the availability of the entire system is calculated. The selection process is carried out using the steady-state selection method, during which the 4 solutions with the highest system availability value are selected from a population of 16 solutions. The selected solutions are the parents of the new generation, which is created by one-point crossover. To avoid the traps of local extremes, the algorithm performs mutation of offspring chromosomes with a probability assumed equal to 0.1. In this way, a new generation is created and the subsequent processes of the genetic algorithm are repeated until a stopping criterion defined as 100 generations is reached.

The flowchart of the genetic algorithm is presented in Fig. 3, and the parameter values are listed in Table 1.

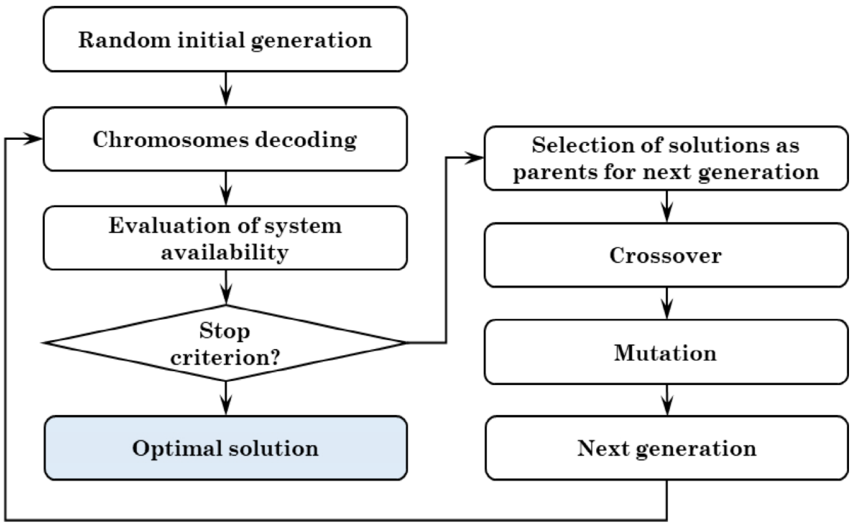

Fig. 3. A flowchart of the developed genetic algorithm.

Table 1. Parameters and methods for the Genetic Algorithm.

Parameters/methods	Values
Number of generations	100
Population size	16
Selection	Steady state selection
Selected solutions for crossover	4
Crossover method	One-point crossover
Mutation probability	0.1

2.6. Availability importance measure-based algorithm

Birnbaum introduced the reliability importance measure to identify which components have the greatest impact on the decrease in system reliability. Birnbaum Importance Measure (BIM) of the ***i***-th component is defined as a ratio of system reliability to component reliability at time ***t*** (Birnbaum ZW, 1969):

$$I_i^B(t) = \frac{\partial R_S(t)}{\partial R_i(t)}. \tag{14}$$

For repairable technical systems, the significance of components (or subsystems) is determined using the availability importance measure (AIM) (Gravette and Barker, 2015):

$$I_i^A = \frac{\partial A_S}{\partial A_i}. \tag{15}$$

AIM, unlike BIM, takes on time-independent values, a property that allows it to be used to determine the importance of components in a system regardless of the assumed lifetime of the system. For the three considered reliability structures, the importance of their individual components has been determined in accordance with the dependencies (16) – (26):

- Series system

$$I_i^A = \frac{\partial A_S}{\partial A_i} = \prod_{j \neq i} A_j, \tag{16}$$

- Series-parallel system

$$I_1^A = \frac{\partial A_S}{\partial A_1} = A_2(1 - (A_3 + A_4 - A_3A_4)A_5), \tag{17}$$

$$I_2^A = \frac{\partial A_S}{\partial A_2} = A_1(1 - (A_3 + A_4 - A_3A_4)A_5), \tag{18}$$

$$I_3^A = \frac{\partial A_S}{\partial A_3} = (A_1A_2 - 1)(A_4 - 1)A_5, \tag{19}$$

$$I_4^A = \frac{\partial A_S}{\partial A_4} = (A_1A_2 - 1)(A_3 - 1)A_5, \tag{20}$$

$$I_5^A = \frac{\partial A_S}{\partial A_5} = (A_1A_2 - 1)(A_3A_4 - A_3 - A_4), \tag{21}$$

- Complex bridge system

$$I_1^A = \frac{\partial A_S}{\partial A_1} = A_2 + A_4A_5 - A_2A_3A_4 - A_2A_3A_5 - A_2A_4A_5 - A_3A_4A_5 + 2A_2A_3A_4A_5, \tag{22}$$

$$I_2^A = \frac{\partial A_S}{\partial A_2} = A_1 + A_3A_5 - A_1A_3A_4 - A_1A_3A_5 - A_1A_4A_5 - A_3A_4A_5 + 2A_1A_3A_4A_5, \tag{23}$$

$$I_3^A = \frac{\partial A_S}{\partial A_3} = A_4 + A_2A_5 - A_1A_2A_4 - A_1A_2A_5 - A_1A_4A_5 - A_2A_4A_5 + 2A_1A_2A_4A_5, \tag{24}$$

$$I_4^A = \frac{\partial A_S}{\partial A_4} = A_3 + A_1A_5 - A_1A_2A_3 - A_1A_2A_5 - A_1A_3A_5 - A_2A_3A_5 + 2A_1A_2A_3A_5, \tag{25}$$

$$I_5^A = \frac{\partial A_S}{\partial A_5} = A_1A_4 + A_2A_3 - A_1A_2A_3 - A_1A_2A_4 - A_1A_3A_4 - A_2A_3A_4 + 2A_1A_2A_3A_4. \tag{26}$$

In addition, in the system design process, it is important to estimate the increase in the subsystem's availability indicator after adding one standby component and the associated cost. The high value of the AIM and the increased availability of the subsystem is a factor prompting the allocation of redundancy in the subsystem, while the cost of the additional component is a factor counteracting such action. Based on the above considerations and theses, an indicator Z_i was proposed to determine the priority in the process of allocating standby components for the i-th subsystem, which is expressed as a relationship:

$$Z_i = \frac{I_i^A(A_i(n_i+1) - A_i(n_i))}{c_i}. \quad (27)$$

Flowchart of iterative algorithm based on availability importance measure is presented in Fig. 4. The first step is to allocate active components to the system. Next, the cost of the system is calculated, and if the optimization cost constraint is met, the algorithm creates models based on continuous-time Markov chains to determine the values of subsystem availability indicators. The system availability and AIM are then calculated based on these subsystem availabilities. The allocation of a standby component to a subsystem is performed according to the largest value of the Z_i indicator, whereby the unit cost of this component cannot exceed the difference between the cost constraint C and the current system cost level. These steps are repeated as long as the difference is greater than or equal to the unit cost of the cheapest component. Thus, the number of iterations of the algorithm is not predetermined.

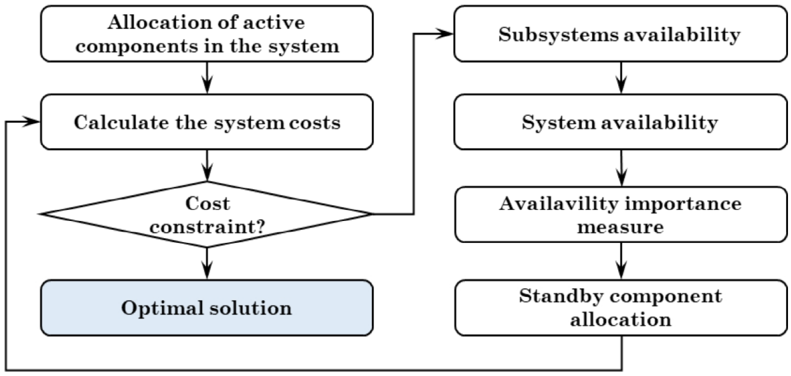

Fig. 4. A flowchart of the availability importance measure-based iterative algorithm.

3. Results

In the conducted research, systems consisting of five active components were considered. Each active component, with its standby components, creates a 1-out-of-n subsystem. Failure and repair rates of the active components are listed in Table 2. Components with lower failure rates were assumed to have lower repair rates, resulting in higher expected repair times. This assumption characterizes the situation that a component with a more advanced and technologically complex design is able to operate for a longer time without failure, however, it also requires a longer repair time. It is further assumed that such a component has a higher unit cost c_i. Three variants of cost constraints were considered, equal to respectively: $C = 20$, $C = 25$ and $C = 30$.

Table 2. Parameters of the components.

Component	Failure rate λ_i	Repair rate μ_i	Unit cost c_i
C1	0.05	0.30	3.50
C2	0.10	0.40	3.00
C3	0.20	0.50	2.00
C4	0.30	0.80	1.50
C5	0.70	1.00	1.00

The number of iterations of the genetic algorithm performed is the same as the number of generations determined as one of the parameters. Too low a number of generations with a low population size may result in achieving a solution far from the optimum. Fig. 5 presents the values of the best RAP solution achieved in successive generations. For the series system (Fig. 5a), with a cost constraint of $C = 20$ and $C = 25$, the genetic algorithm found the optimal solutions immediately. Whereas for $C = 30$, after an initial significant increase in the value of the objective function, another increase occurred in the 68th generation. Different results were obtained for the series-parallel system. According to Fig. 5b, at the latest 59 generations, the algorithm found the optimal solution for the variant $C = 20$. The complex bridge system proved to be the most difficult problem (Fig. 5c). The genetic algorithm gradually approached optimal solutions for all three cost constraints.

The number of iterations of the AIM-based algorithm depends on the number of standby components allocated in the system. Each iteration corresponds to one such component. Fig. 6 presents the computational results of the optimization problem achieved using the developed AIM-based algorithm. The trajectory of the value of the objective function is monotonically increasing due to the successive enlargement of the system structure. An increase in the value of the cost constraint C results in a greater number of iterations performed by the algorithm. For the $C = 20$ constraint, AIM-based algorithm for solving the RAP performed 5 iterations in series system and series parallel system and 6 iterations for complex bridge system. In contrast, for $C = 25$ and $C = 30$, the number of iterations performed ranged from 6 to 9 and 7 to 12, respectively.

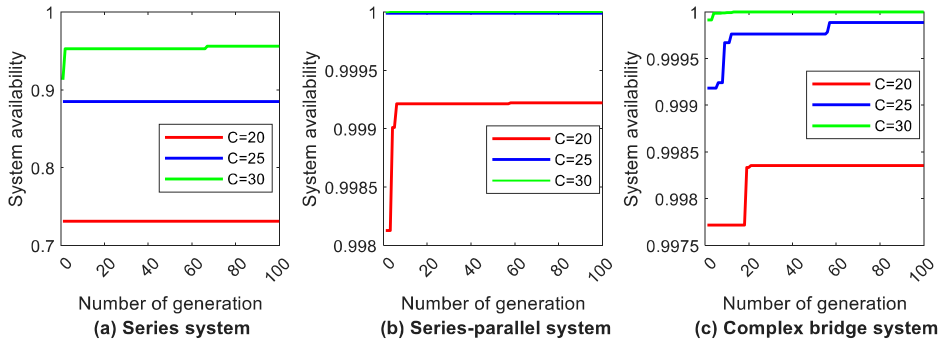

Fig. 5. Results of the developed GA for: (a) series system, (b) series-parallel system, (c) complex bridge system.

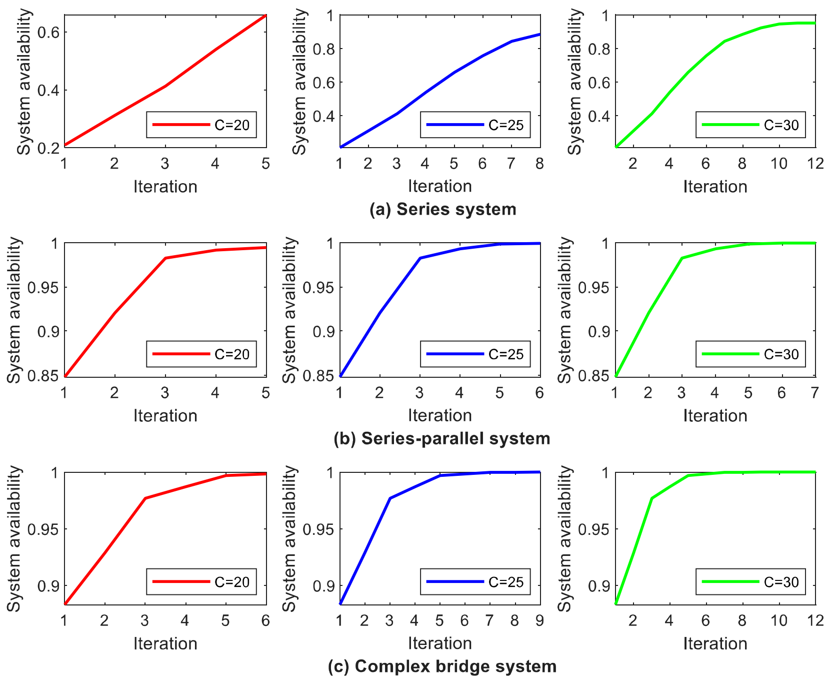

Fig. 6. Results of the developed AIM-based algorithm for: (a) series system, (b) series-parallel system, (c) complex bridge system.

A comparison of the results obtained with the two algorithms is summarized in Table 3. For the series system at $C = 20$ and $C = 25$, as well as the complex bridge system at $C = 30$, both the GA and AIM-based algorithm discovered identical solutions that are assumed to be optimal. In five instances, GA obtained better efficiency by finding a solution with a higher system availability value. Nevertheless, in one instance, for a complex bridge system and $C = 25$, the AIM-based algorithm was more effective. However, it should be noted that the differences in the values of the objective function were low. On the other hand, the AIM-based algorithm required about 36 times less computational time than GA for all cases considered. This comparison presents a significant advantage in computational performance of the proposed approach.

Table 3. Results of availability optimization in the RAP.

System	Cost constraint	Optimization method	System configuration	System availability	System cost	Computation time (s)
Series	C = 20	GA	1 – 2 – 2 – 2 – 3	0.73106499	19.50	0.8906
		AIM	1 – 2 – 2 – 2 – 3	0.73106499	19.50	0.0938
	C = 25	GA	2 – 2 – 3 – 2 – 3	0.88466325	25.00	1.0938
		AIM	2 – 2 – 3 – 2 – 3	0.88466325	25.00	0.0625
	C = 30	GA	2 – 2 – 3 – 4 – 5	0.95602546	30.00	1.2656
		AIM	2 – 2 – 4 – 3 – 4	0.95285811	29.50	0.0156
Series - parallel	C = 20	GA	1 – 1 – 1 – 4 – 5	0.99922198	19.50	1.6406
		AIM	2 – 2 – 1 – 1 – 3	0.99470928	19.50	0.0156
	C = 25	GA	1 – 1 – 5 – 1 – 7	0.99998975	25.00	1.3281
		AIM	3 – 3 – 1 – 1 – 2	0.99937553	25.00	0.0156
	C = 30	GA	1 – 1 – 1 – 8 – 9	0.99999988	30.00	1.1406
		AIM	3 – 5 – 1 – 1 – 1	0.99968094	30.00	0.0156
Complex bridge	C = 20	GA	1 – 1 – 4 – 3 – 1	0.99835470	20.00	0.9688
		AIM	1 – 1 – 3 – 4 – 1	0.99833334	19.50	0.0313
	C = 25	GA	1 – 1 – 4 – 5 – 3	0.99988572	25.00	1.1094
		AIM	1 – 1 – 5 – 5 – 1	0.99997730	25.00	0.0313
	C = 30	GA	1 – 1 – 6 – 7 – 1	0.99999915	30.00	1.3438
		AIM	1 – 1 – 6 – 7 – 1	0.99999915	30.00	0.0156

4. Conclusions

In summary, the paper presents two approaches to solving the Redundancy Allocation Problem. The three presented systems are widely used in the scientific literature to validate proposed approaches and methods for maximizing reliability and availability under an assumed cost constraint. The obtained results confirm the usefulness of both algorithms. The meta-heuristic genetic algorithm reaches high efficiency in finding solutions to maximize system availability. However, it requires significantly longer computation time compared to the AIM-based iterative algorithm. Genetic algorithm requires defining basic rules and parameters for selection, crossover and mutation. The randomization of initial solutions and the stochastic nature of mutation results in reaching a different way of finding the optimal solution each time. In contrast to the GA, the AIM-based algorithm is determined without stochastic components. For this reason, the course of changes in the value of the objective function in successive iterations is always the same for given assumptions and parameters.

In future studies, several other assumptions are worth considering. Firstly, the other standby modes (warm and hot) should be investigated. This is related to the Markov model, which is the basis for calculating subsystem availability values. Secondly, the distributions of times to failure and repair times of components should be generalized by developing a semi-Markov model. Finally, testing the developed algorithms on large-scale systems can reveal other properties, including the advantages and limitations of the proposed approaches.

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